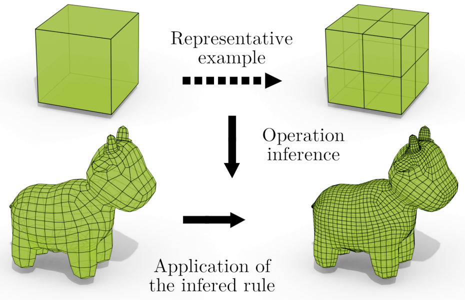
Inference of graph transformation rules for the design of geometric modeling operations



Romain Pascual

under the supervision of
Pascale Le Gall, Hakim Belhaouari,
and Agnès Arnould

November 29, 2022

Geometric modeling

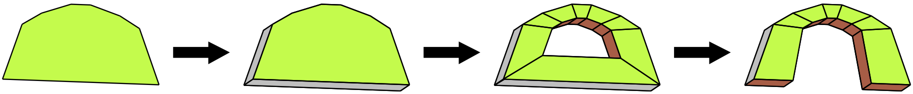
- How to realize such a scene?

Geometric modeling

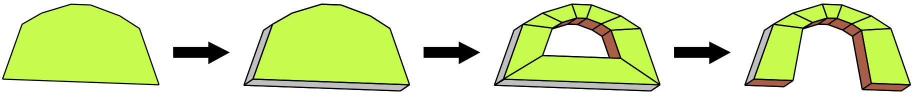
- How to realize such a scene?

Creation of the objects, displacement mapping, normal mapping, 3D fractal texture, rendering, ...

Creating objects

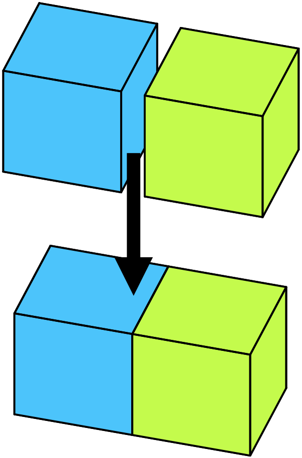


Creating objects

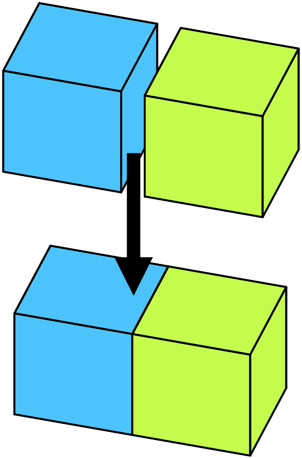


Diversity of tools: Blender, AutoCAD, Catia, Houdini, Maya, Rhino 3D, SketchUp, SolidWorks, . . .

Designing modeling operations



Designing modeling operations

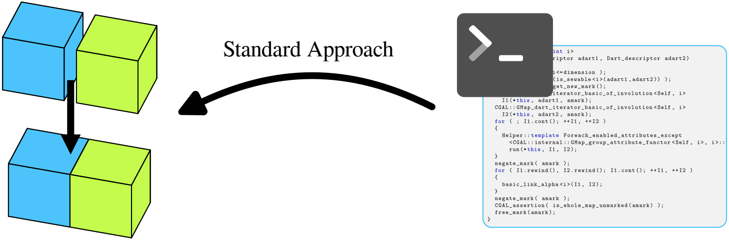


CGAL's sew operation

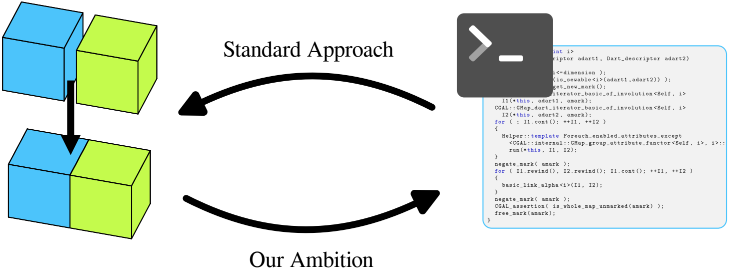
```
template<unsigned int i>
void sew(Dart_descriptor adart1, Dart_descriptor adart2)

  CGAL_assertion( i<=dimension );
  CGAL_assertion( (is_sewable<i>(adart1,adart2)) );
  size_type amark=get_new_mark();
  CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
    I1(*this, adart1, amark);
  CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
    I2(*this, adart2, amark);
  for ( ; I1.cont(); ++I1, ++I2 )
  {
    Helper::template Foreach_enabled_attributes_except
      <CGAL::internal::GMap_group_attribute_functor<Self, i>, i>::
      run(*this, I1, I2);
  }
  negate_mark( amark );
  for ( I1.rewind(), I2.rewind(); I1.cont(); ++I1, ++I2 )
  {
    basic_link_alpha<i>(I1, I2);
  }
  negate_mark( amark );
  CGAL_assertion( is_whole_map_unmarked(amark) );
  free_mark(amark);
}
```

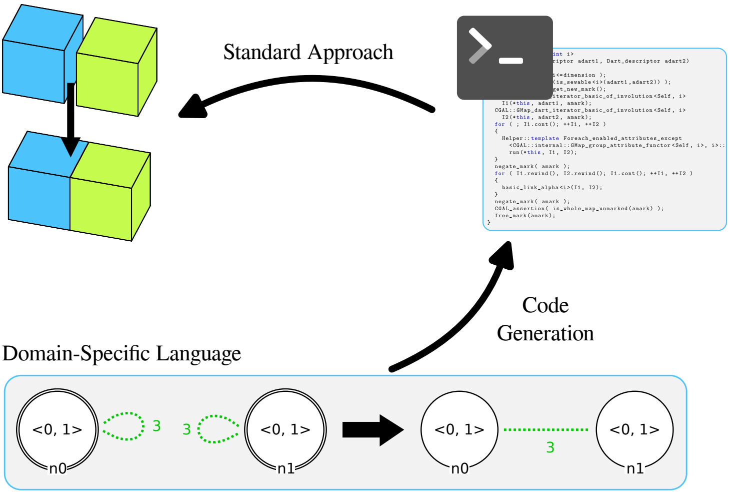
Inferring modeling operations



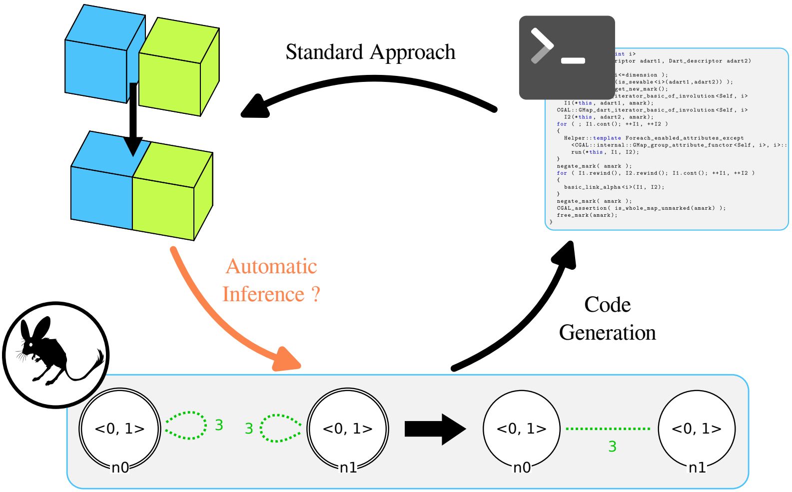
Inferring modeling operations



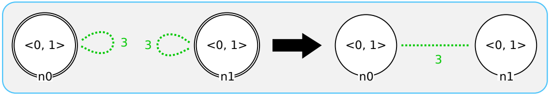
Inferring modeling operations



Inferring modeling operations

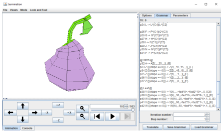
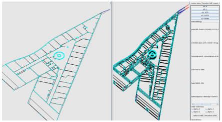
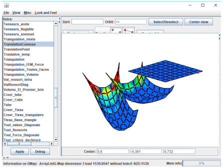
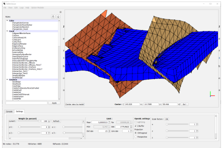


Strengths and weaknesses of Jerboa's DSL



- Main characteristics:
  - Dedicated to Gmaps[1]
  - Syntax analyzer exploiting sufficient conditions[2]

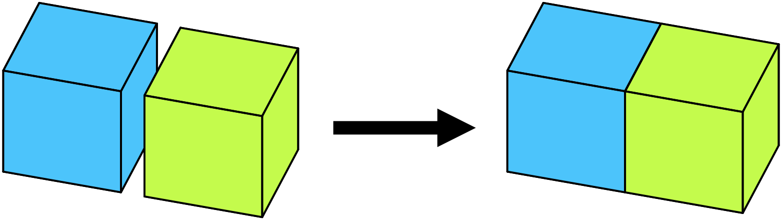
- Successful applications:
  - Plant growth
  - Architecture
  - Spring-mass
  - Geology

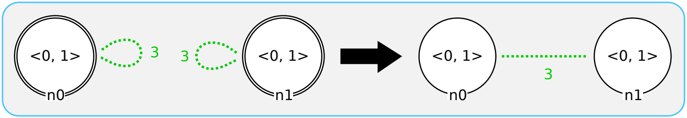





[1] Poudret et al. 2008.

[2] Belhaouari et al. 2014.

A sneak peek at Jerboa's language





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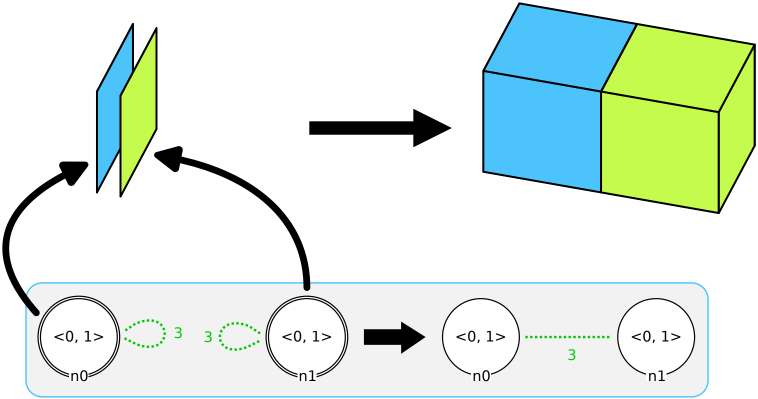

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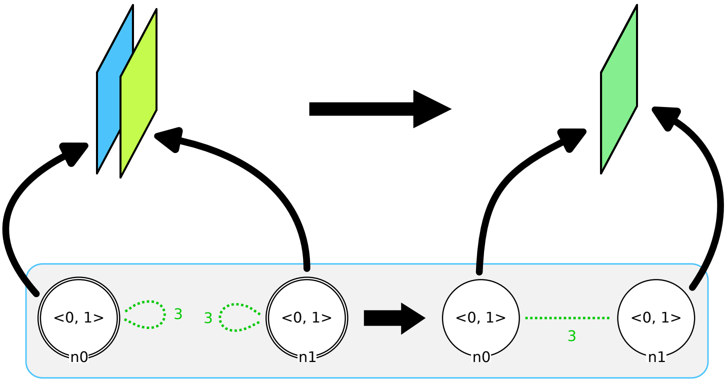
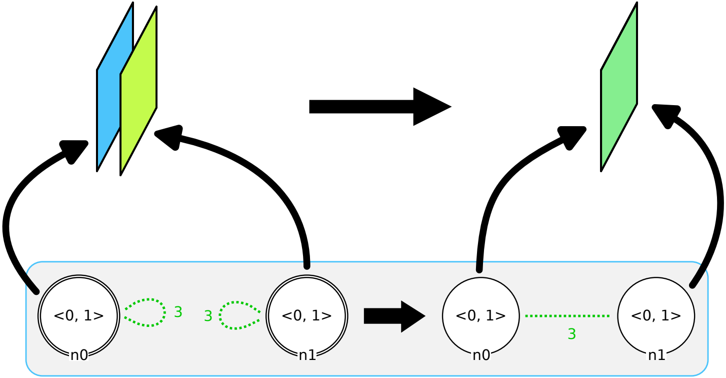


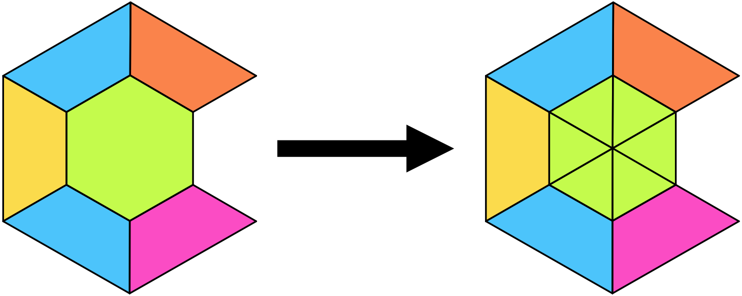
A sneak peek at Jerboa's language



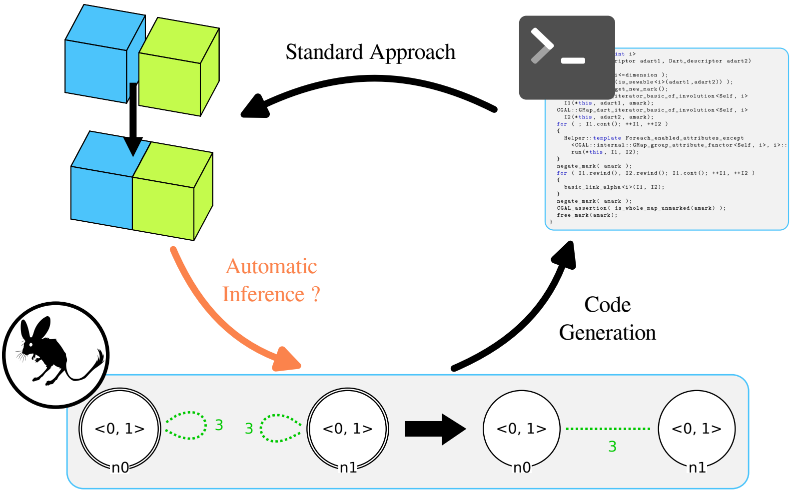
- Double-pushout (DPO) approach to graph transformations.[1]

[1] Rozenberg 1997; Ehrig et al. 2006; Heckel et al. 2020.

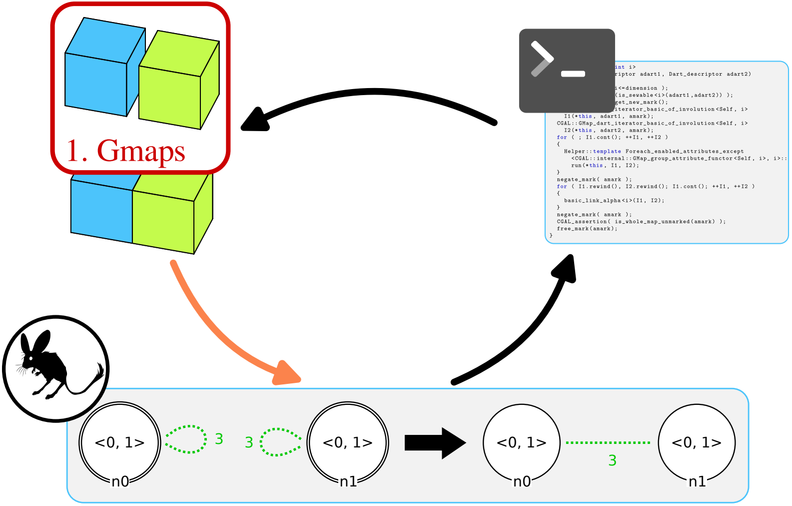
Running example: face triangulation



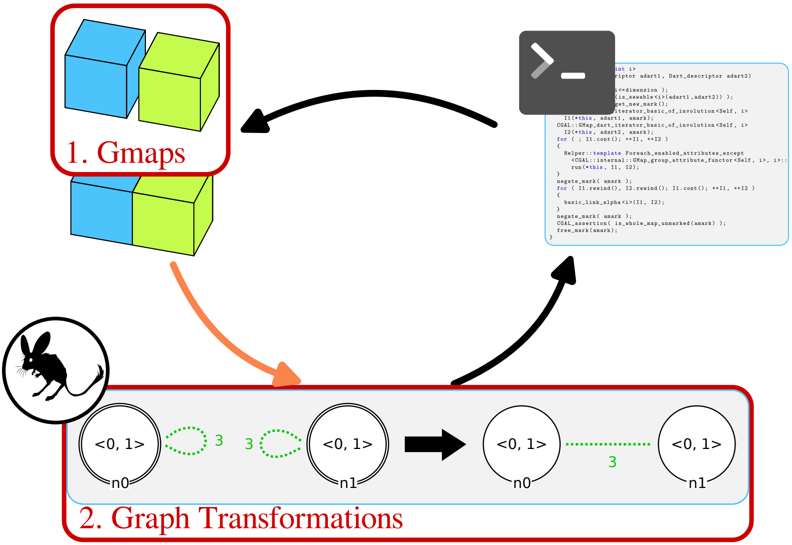
Plan


Standard Approach
Automatic Inference ?
Code Generation
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1

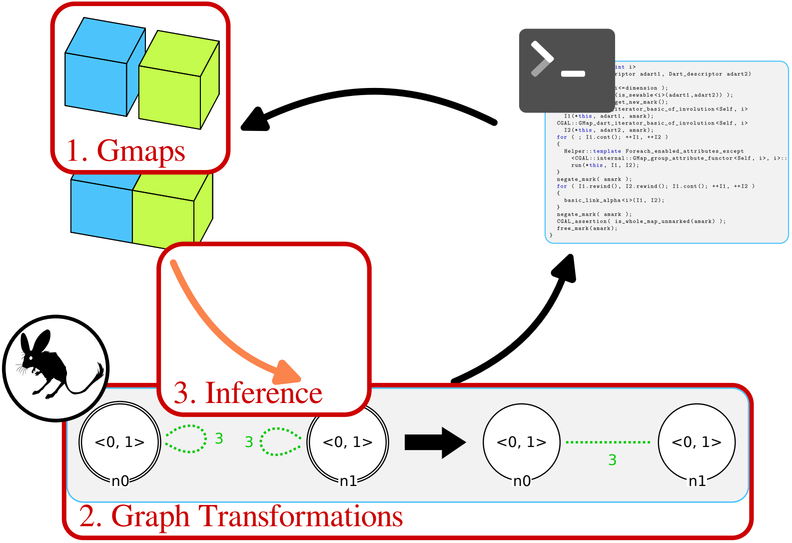
Plan



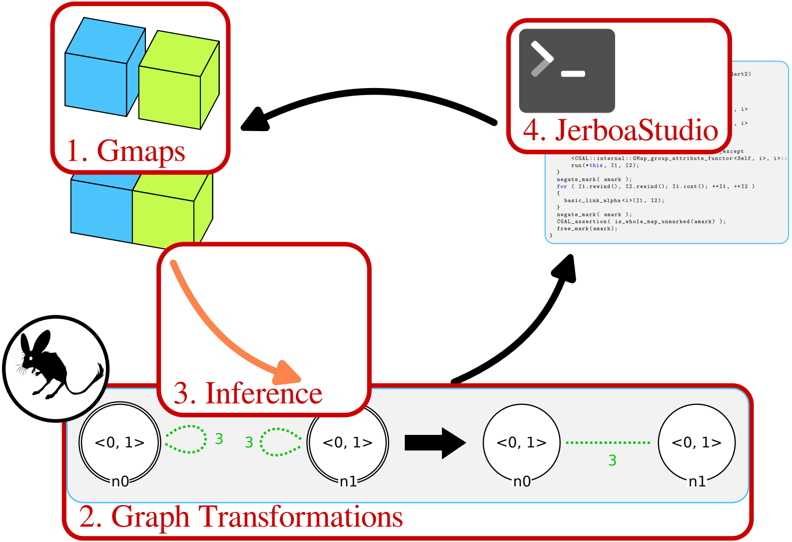
Plan



Plan


1. Gmaps
3. Inference
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1
2. Graph Transformations

Plan


1. Gmaps
4. JerboaStudio
3. Inference
2. Graph Transformations
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1

Generalized maps

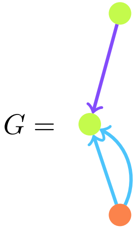
- Geometric objects are represented with embedded generalized maps.

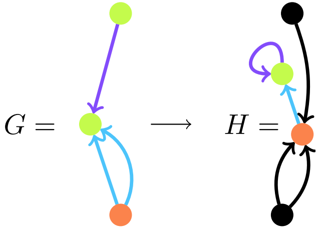
The category of graphs

$G =$

- A graph $G = (V, E, s, t)$:
  - a set of nodes V,
  - a set of arcs E,
  - a source function $s : E \to V$,
  - a target arrow $t : E \to V$,

The category of graphs





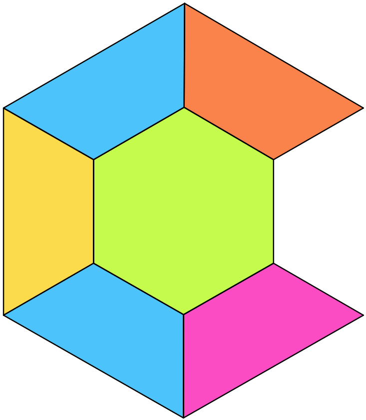
- A graph $G = (V, E, s, t)$:
  - a set of nodes V,
  - a set of arcs E,
  - a source function $s : E \to V$,
  - a target arrow $t : E \to V$,

- A morphism $G \to H$:
  - a node function $V_G \to V_H$,
  - an arc function $E_G \to E_H$,

  preserving structure.

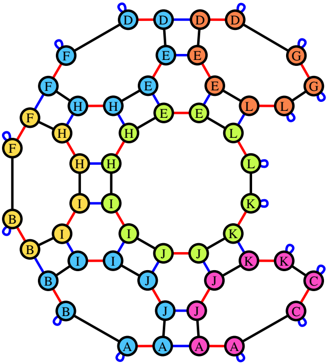
- Graphs can be decorated with labels, types, and attributes.

Generalized maps[1]



[1] Damiand et al. 2014.

Generalized maps[1]

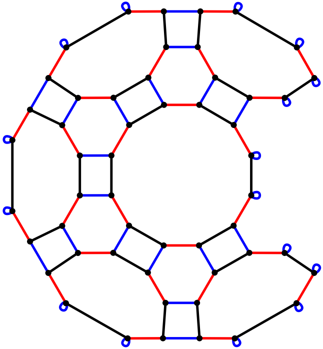


Color legend: 0, 1, 2.

Gmaps built as graphs:

[1] Damiand et al. 2014.

Generalized maps[1]



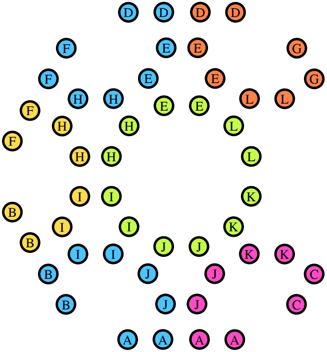
Color legend: 0, 1, 2.

Gmaps built as graphs:

- topology: graph structure

[1] Damiand et al. 2014.

Generalized maps[1]



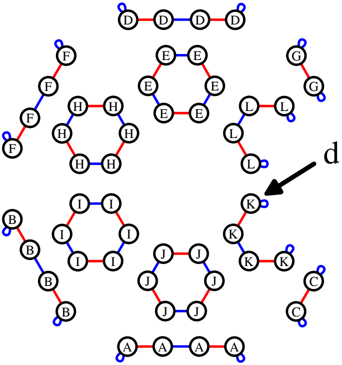
Color legend: 0, 1, 2.

Gmaps built as graphs:

- topology: graph structure
- geometry: node attributes

[1] Damiand et al. 2014.

Orbits and topological cells

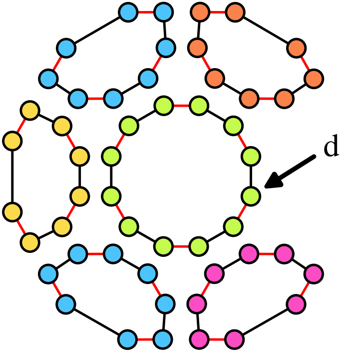


Color legend: 0, 1, 2.

▶ Orbit (encode topological cell):
Graph induced by a subset $\langle o \rangle \subseteq [\![0, n]\!]$ of dimensions.

- positions on vertices (orbits $\langle 1, 2 \rangle$).

Orbits and topological cells



▶ Orbit (encode topological cell):
Graph induced by a subset $\langle o \rangle \subseteq [\![0, n]\!]$ of dimensions.

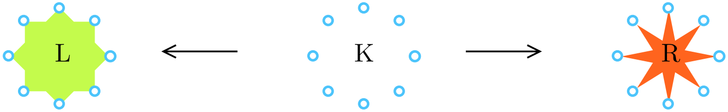
- positions on vertices (orbits $\langle 1, 2 \rangle$).
- colors on faces (orbits $\langle 0, 1 \rangle$).

Color legend: 0, 1, 2.

Graph rewriting

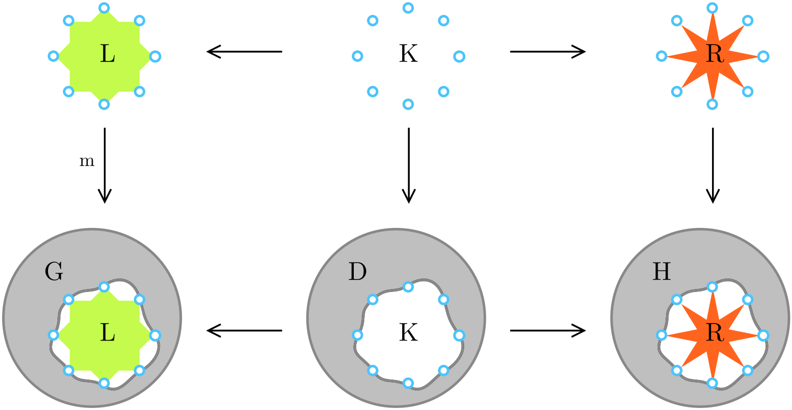
- Operations on Gmaps are designed as graph rewriting rules.

Graph transformation rules[1]



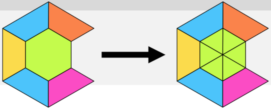
[1] Rozenberg 1997; Ehrig et al. 2006; Heckel et al. 2020.

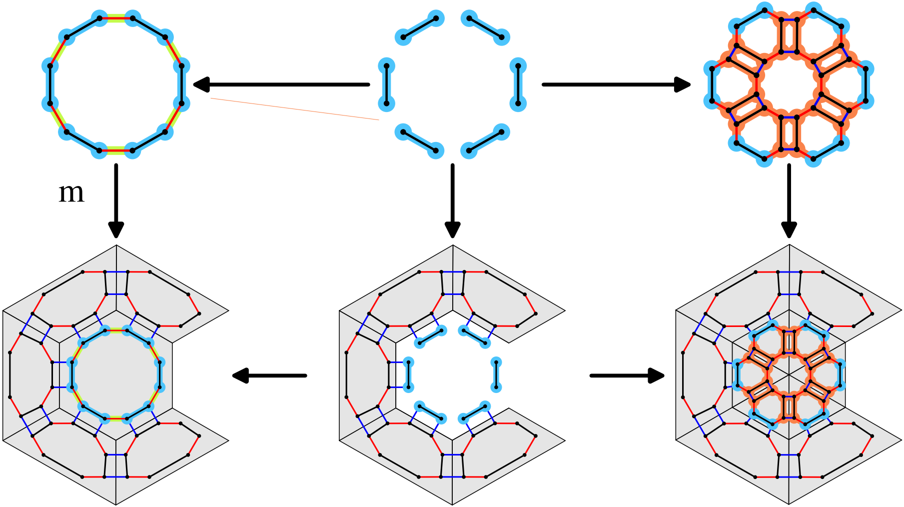
Graph transformation rules[1]



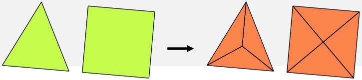
[1]Rozenberg 1997; Ehrig et al. 2006; Heckel et al. 2020.

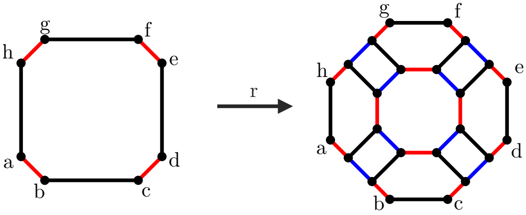
Rewriting Gmaps





Orbit rewriting





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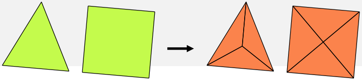
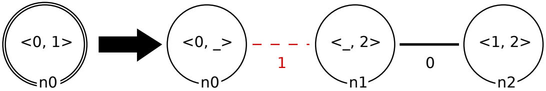
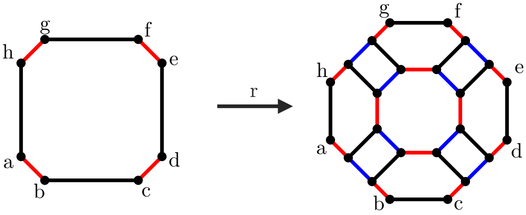
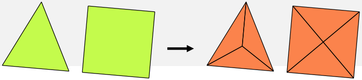
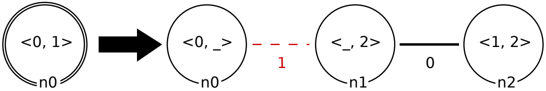


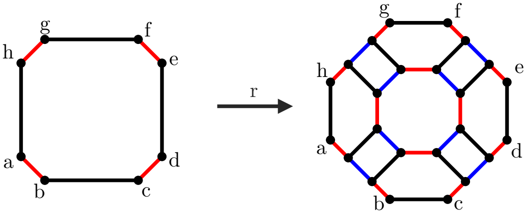




Orbit rewriting







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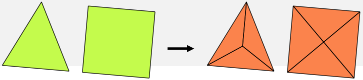

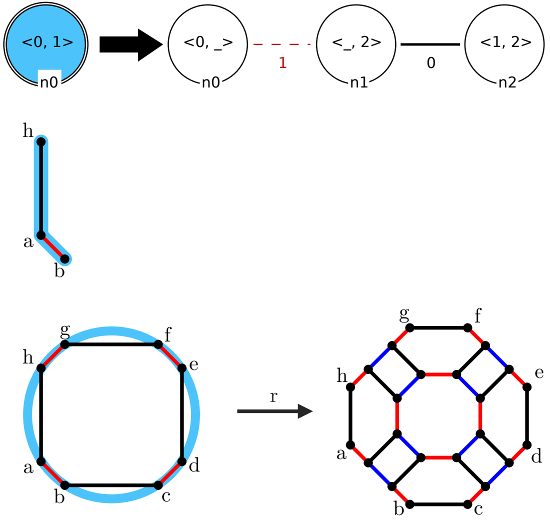

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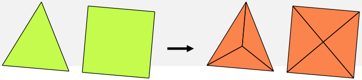

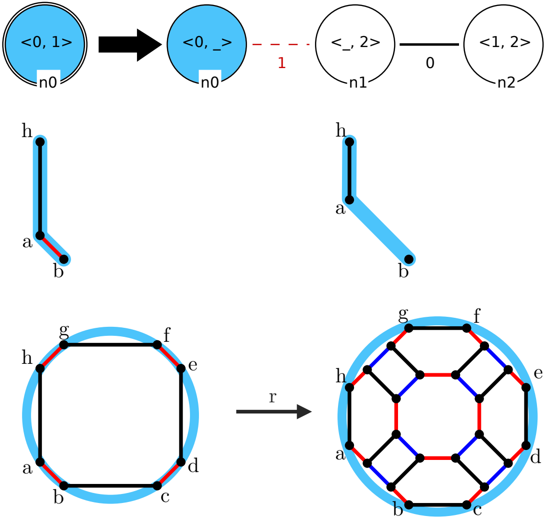

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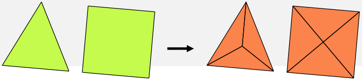

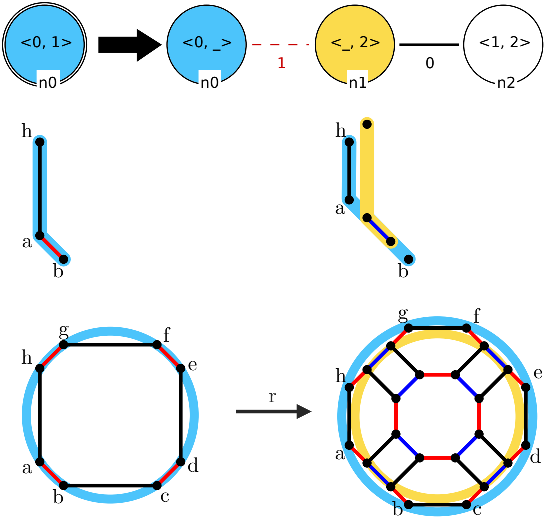

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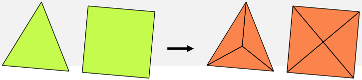

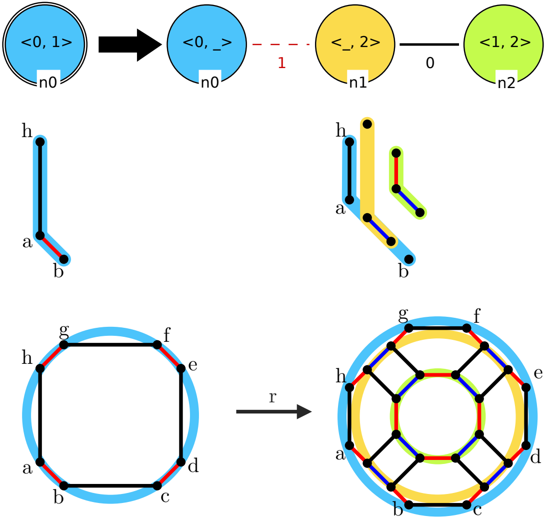

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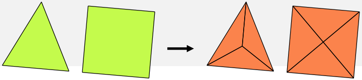

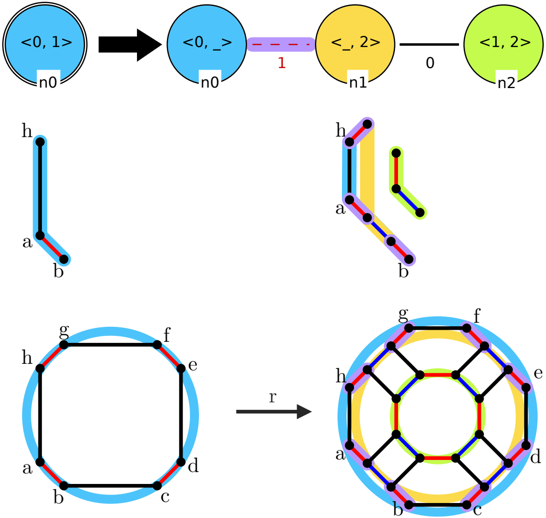

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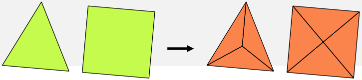

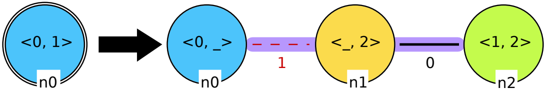

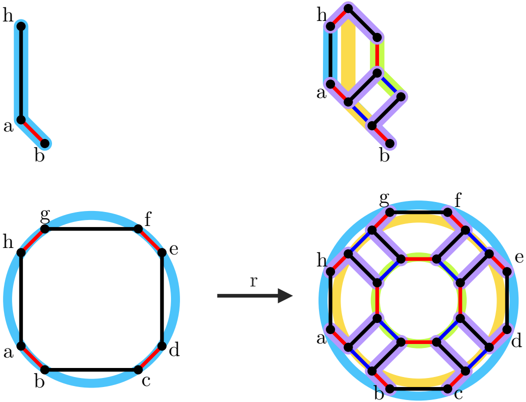

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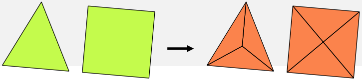
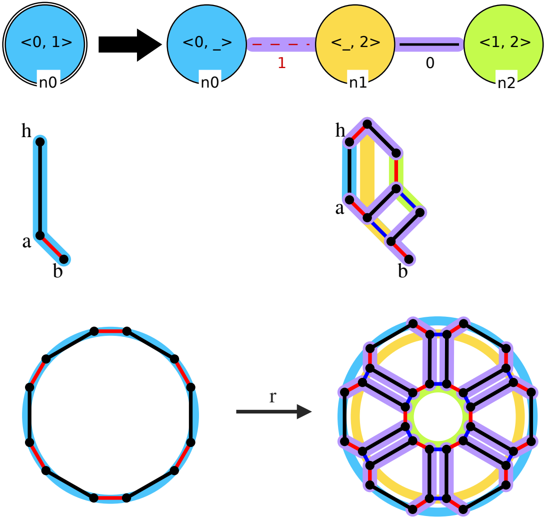
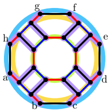
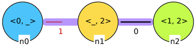


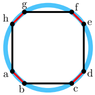


Graph products[1]

- A categorical construction of global relabeling



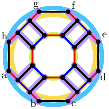


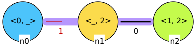


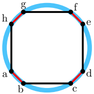
[1] inspired from Bauderon 1995.

Graph products[1]

- A categorical construction of global relabeling





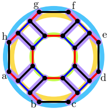


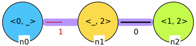
P

[1] inspired from Bauderon 1995.

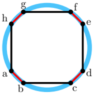
Graph products[1]

- A categorical construction of global relabeling





Π



P

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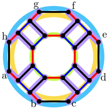
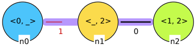
Graph products[1]

- A categorical construction of global relabeling



$\iota(\Pi, P)$



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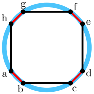


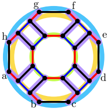
P

- $\iota(\Pi, P)$: instantiation.

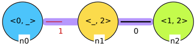
[1] inspired from Bauderon 1995.

Graph products[1]

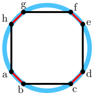
- A categorical construction of global relabeling



$\iota(\Pi, P)$



Π



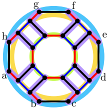
$$P \xRightarrow{\mathbb{E}_\Sigma} \mathbb{E}_\Sigma(P)$$

- $\iota(\Pi, P)$: instantiation.
- $\mathbb{E}_\Sigma$: embedding functor.

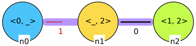
[1] inspired from Bauderon 1995.

Graph products[1]

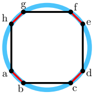
- A categorical construction of global relabeling



$\iota(\Pi, P)$



$$\begin{tikzcd} & \mathbb{E}_\Sigma(P) \times \Pi \arrow[r] \arrow[d] \arrow[dr, phantom, "PB"] & \Pi \arrow[d, "!_\Pi"] \\ P \arrow[r, Rightarrow, "\mathbb{E}_\Sigma"] & \mathbb{E}_\Sigma(P) \arrow[r, "!_{\mathbb{E}_\Sigma(P)}"] & 1_{\Sigma^2} \end{tikzcd}$$

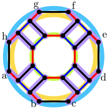


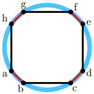
- $\iota(\Pi, P)$: instantiation.
- $\mathbb{E}_\Sigma$: embedding functor.

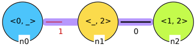
[1] inspired from Bauderon 1995.

Graph products[1]

▶ A categorical construction of global relabeling







$$
\begin{tikzcd}
\color{red}{\iota(\Pi, P)} & \mathbb{E}_\Sigma(P) \times \Pi \arrow[l, Rightarrow, "\pi_\Sigma"'] \arrow[r] \arrow[d] \arrow[dr, phantom, "PB"] & \color{red}{\Pi} \arrow[d, "!_\Pi"] \\
\color{red}{P} \arrow[r, Rightarrow, "\mathbb{E}_\Sigma"] & \mathbb{E}_\Sigma(P) \arrow[r, "!_{\mathbb{E}_\Sigma(P)}"] & 1_{\Sigma^2}
\end{tikzcd}
$$

- $\iota(\Pi, P)$: instantiation.
- $\mathbb{E}_\Sigma$: embedding functor.
- π_Σ: projecting functor.

[1] inspired from Bauderon 1995.

Application of a rule scheme

$$
\begin{tikzcd}
1_{\mathbb{W}\times\mathbb{D}} & & & \\
E_{\mathbb{D}}(P_v) \arrow[u, "!_{E_{\mathbb{D}}(P_v)}"] & L \arrow[ul, "!_L"'] & K \arrow[l, hook'] \arrow[r, hook] \arrow[ull, "!_K"'] & R \arrow[ulll, "!_R"'] \\
P_v \arrow[u, Rightarrow, "E_{\mathbb{D}}"] \arrow[d, hook, "p_v"'] & E_{\mathbb{D}}(P_v) \times L \arrow[u] \arrow[ul] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v) \times K \arrow[l, hook'] \arrow[r, hook] \arrow[u] \arrow[ull] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v) \times R \arrow[u] \arrow[ulll] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] \\
\mathbb{P}(\mathcal{G}) & \iota(L, P_v) \arrow[d, hook, "m"] & \iota(K, P_v) \arrow[l, hook'] \arrow[r, hook] \arrow[d, hook] & \iota(R, P_v) \arrow[d, hook] \\
 & \mathcal{G} \arrow[ul, Rightarrow, "\mathbb{P}"] & \mathcal{D} \arrow[l, hook'] \arrow[r, hook] & \mathcal{H}
\end{tikzcd}
$$

Application of a rule scheme

$$
\begin{tikzcd}
1_{\mathbb{W}\times\mathbb{D}} & & & \\
E_{\mathbb{D}}(P_v) \arrow[u, "!_{E_{\mathbb{D}}(P_v)}"] & L \arrow[ul, "!_L"'] & K \arrow[l, hook'] \arrow[r, hook] \arrow[ull, "!_K"'] & R \arrow[ulll, "!_R"'] \\
P_v \arrow[u, Rightarrow, "E_{\mathbb{D}}"] \arrow[d, hook, "p_v"'] & E_{\mathbb{D}}(P_v)\times L \arrow[ul] \arrow[u, dash] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v)\times K \arrow[l, hook'] \arrow[r, hook] \arrow[ull] \arrow[u, dash] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v)\times R \arrow[ulll] \arrow[u, dash] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] \\
\mathbb{P}(\mathcal{G}) & \iota(L,P_v) \arrow[d, hook, "m"] & \iota(K,P_v) \arrow[l, hook'] \arrow[r, hook] \arrow[d, hook] & \iota(R,P_v) \arrow[d, hook] \\
 & \mathcal{G} \arrow[ul, Rightarrow, "\mathbb{P}"] & \mathcal{D} \arrow[l, hook'] \arrow[r, hook] & \mathcal{H}
\end{tikzcd}
$$

Input 1: Rule scheme

Input 2: Gmap

Output: Gmap

Application of a rule scheme

$$
\begin{tikzcd}
1_{\mathbb{W}\times\mathbb{D}} & & & \\
E_{\mathbb{D}}(P_v) \arrow[u, "!_{E_{\mathbb{D}}(P_v)}"] & \color{red}{L} \arrow[ul, "!_L"'] & \color{red}{K} \arrow[ull, "!_K"'] \arrow[l, hook', red] \arrow[r, hook, red] & \color{red}{R} \arrow[ulll, "!_R"'] \\
P_v \arrow[u, Rightarrow, "E_{\mathbb{D}}"] \arrow[d, "p_v"'] & E_{\mathbb{D}}(P_v) \times L \arrow[ul] \arrow[u] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v) \times K \arrow[ull] \arrow[u] \arrow[l, hook'] \arrow[r, hook] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v) \times R \arrow[ulll] \arrow[u] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] \\
\mathbb{P}(\mathcal{G}) & \iota(L, P_v) \arrow[d, "m"] & \iota(K, P_v) \arrow[l, hook'] \arrow[r, hook] \arrow[d, hook] & \iota(R, P_v) \arrow[d, hook] \\
 & \color{red}{\mathcal{G}} \arrow[ul, Rightarrow, "\mathbb{P}"] & \mathcal{D} \arrow[l, hook'] \arrow[r, hook] & \color{red}{\mathcal{H}}
\end{tikzcd}
$$

Product

Application of a rule scheme

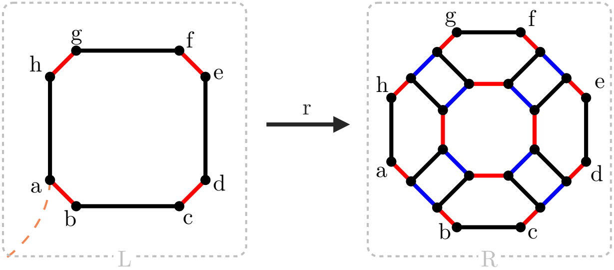
$$
\begin{tikzcd}
1_{\mathbb{W}\times\mathbb{D}} & & & \\
E_{\mathbb{D}}(P_v) \arrow[u, "!_{E_{\mathbb{D}}(P_v)}"] & \color{red}{L} \arrow[ul, "!_L"'] & \color{red}{K} \arrow[l, hook', red] \arrow[r, hook, red] \arrow[ull, "!_K"'] & \color{red}{R} \arrow[ulll, "!_R"'] \\
P_v \arrow[u, Rightarrow, "E_{\mathbb{D}}"] \arrow[d, hook, "p_v"'] & E_{\mathbb{D}}(P_v)\times L \arrow[u] \arrow[ul] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v)\times K \arrow[l, hook'] \arrow[r, hook] \arrow[u] \arrow[ull] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v)\times R \arrow[u] \arrow[ulll] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] \\
\mathbb{P}(\mathcal{G}) & \iota(L,P_v) \arrow[d, hook, "m"] & \iota(K,P_v) \arrow[l, hook'] \arrow[r, hook] \arrow[d, hook] & \iota(R,P_v) \arrow[d, hook] \\
 & \color{red}{\mathcal{G}} \arrow[ul, Rightarrow, "\mathbb{P}"] & \mathcal{D} \arrow[l, hook'] \arrow[r, hook] & \color{red}{\mathcal{H}}
\end{tikzcd}
$$

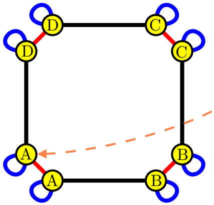
'Standard' DPO step

Modifying geometric values[1]

[1]Bellet et al. 2017.

Modifying geometric values[1]



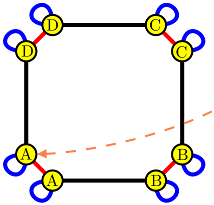


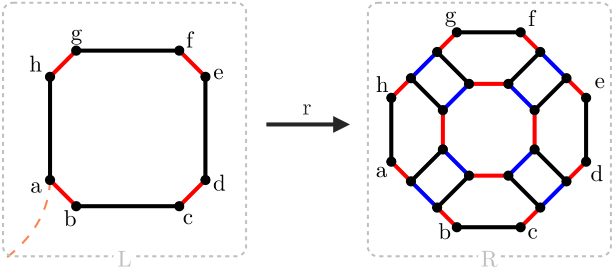
Embedding expressions modeled with algebraic data types:

- add
- middle
- scale
- ...

[1]Bellet et al. 2017.

Modifying geometric values[1]



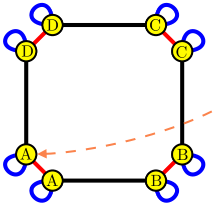


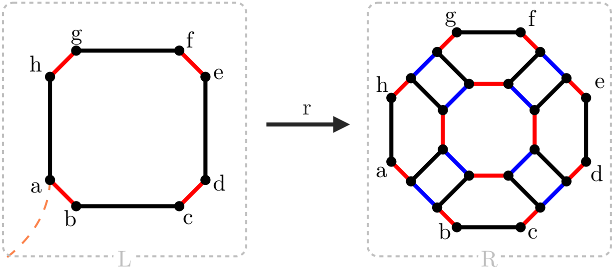
Embedding expressions are extended with topological operators:

- Neighbor operator:
  - $a@0@1@0.position = f.position = C$
  - $a@1@0.color = c.color =$

[1]Bellet et al. 2017.

Modifying geometric values[1]



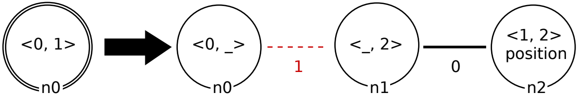


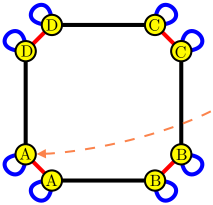
Embedding expressions are extended with topological operators:

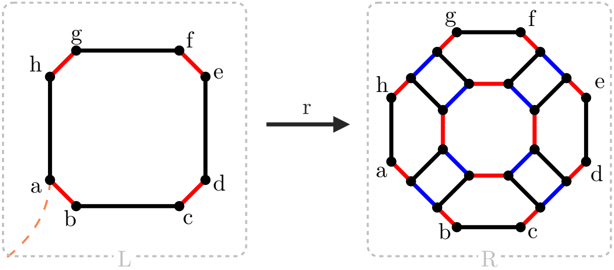
- Neighbor operator:
- Collect operator:
  - $\mathit{position}_{\langle 0,1\rangle}(a) = \{A, B, C, D\}$
  - $\mathit{color}_{\langle 0,1\rangle}(a) = \{\bullet\}$

[1] Bellet et al. 2017.

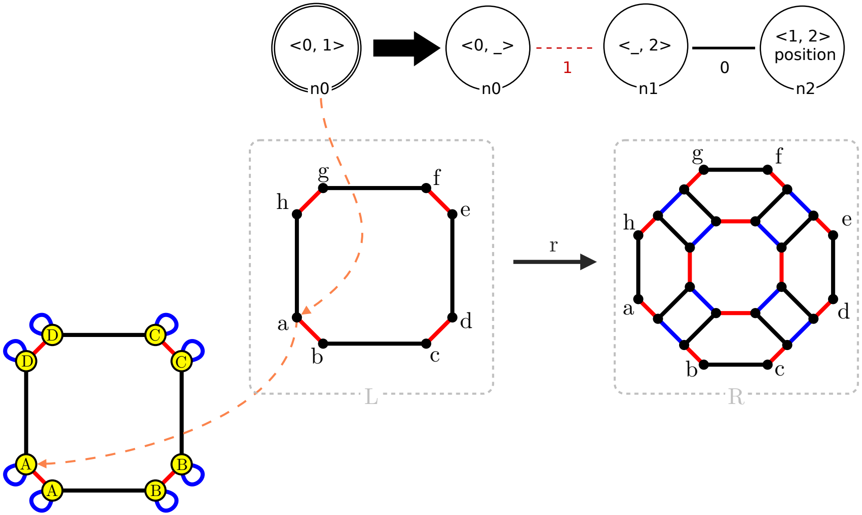
Extension to schemes



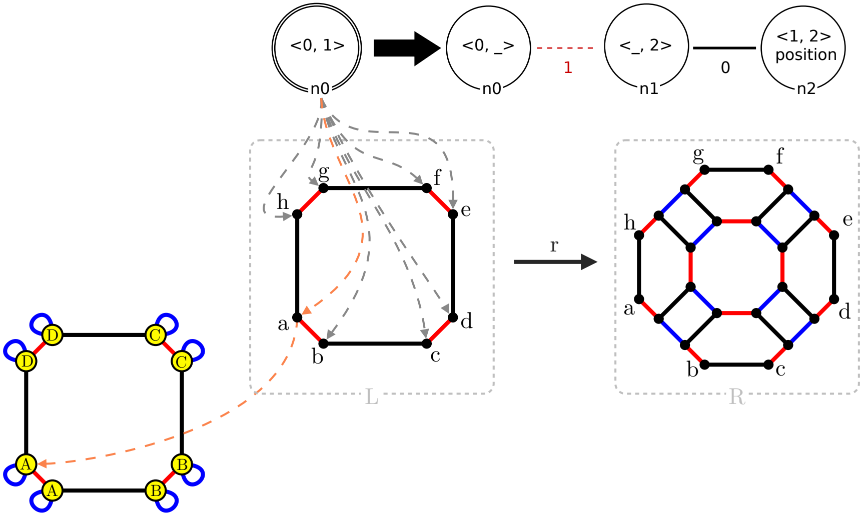




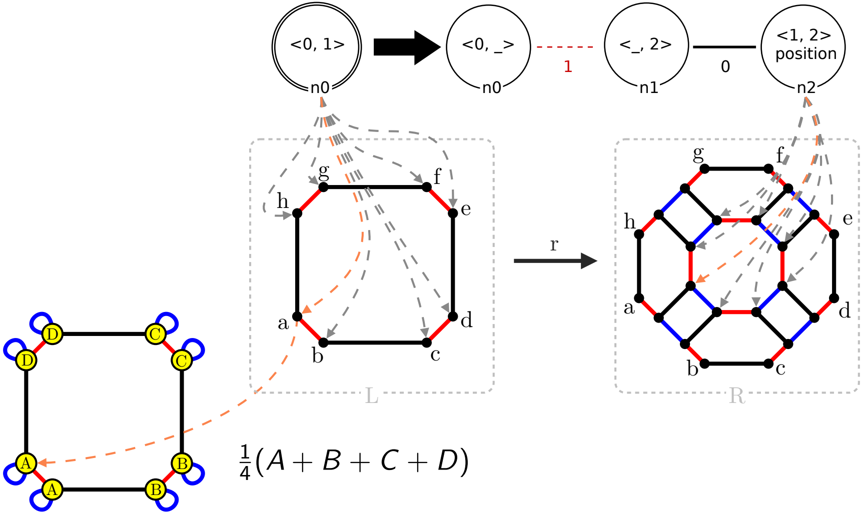
Extension to schemes



Extension to schemes

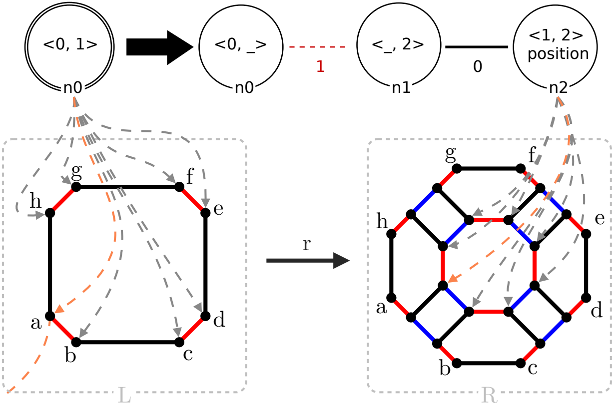


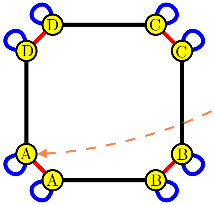
Extension to schemes



$\frac{1}{4}(A + B + C + D)$

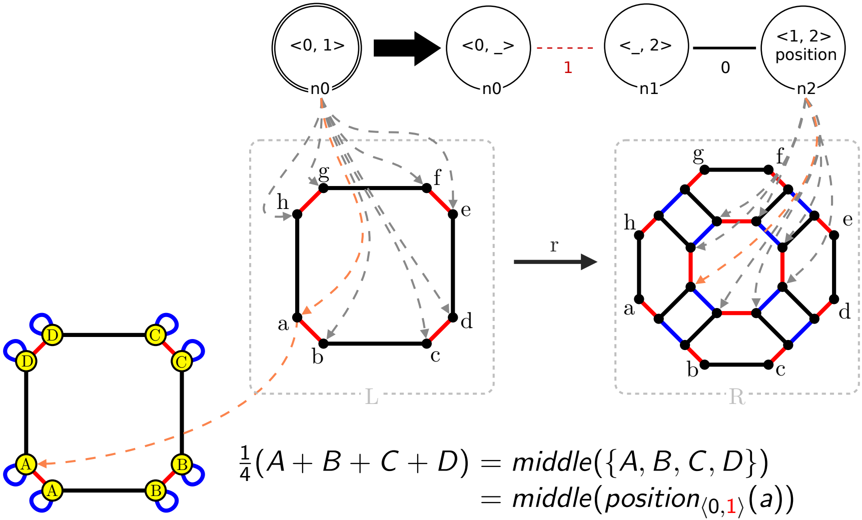
Extension to schemes





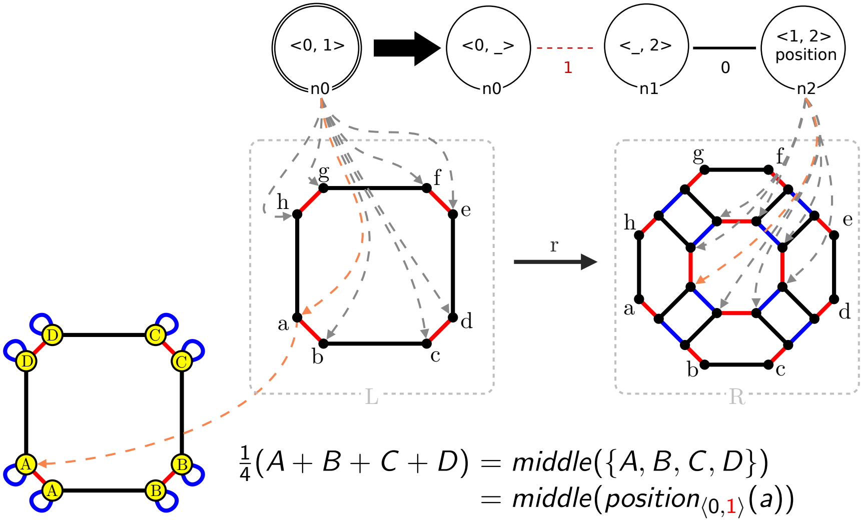
$$\frac{1}{4}(A + B + C + D) = middle(\{A, B, C, D\})$$

Extension to schemes



$$\frac{1}{4}(A+B+C+D) = middle(\{A,B,C,D\})$$
$$= middle(position_{\langle 0,1\rangle}(a))$$

Extension to schemes



$$\frac{1}{4}(A + B + C + D) = middle(\{A, B, C, D\})$$
$$= middle(position_{\langle 0,1\rangle}(a))$$
$$= middle(position_{\langle 0,1\rangle}(n0))$$

Consistency preservation

- Modifications of a well-formed object should produce an equally well-formed object.

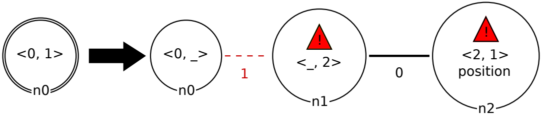
Requirement: Provide feedback to the rule designer.

Consistency preservation

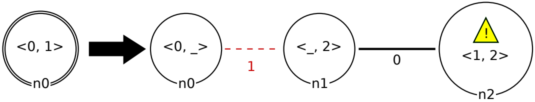
- Modifications of a well-formed object should produce an equally well-formed object.

Requirement: Provide feedback to the rule designer.

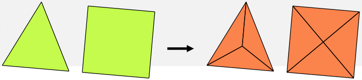
- Topological inconsistency



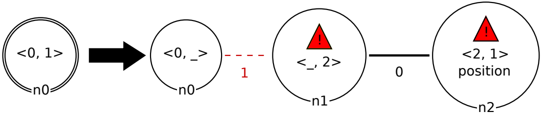
- Geometric inconsistency

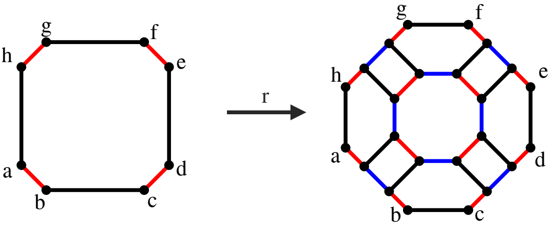


Breaking the topological consistency

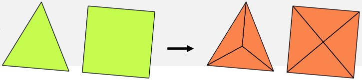


Constraint: 0202 paths should be cycles.





Breaking the topological consistency



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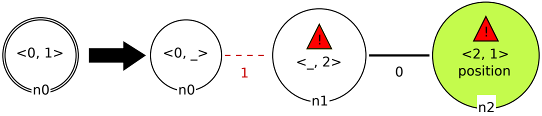
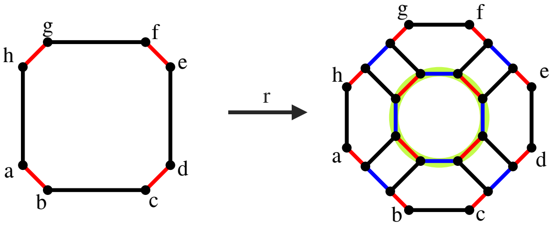
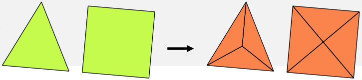




Breaking the topological consistency



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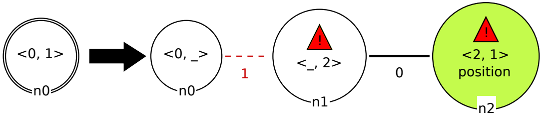
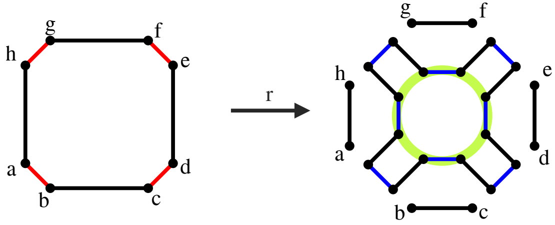
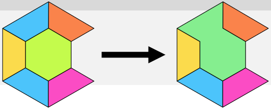
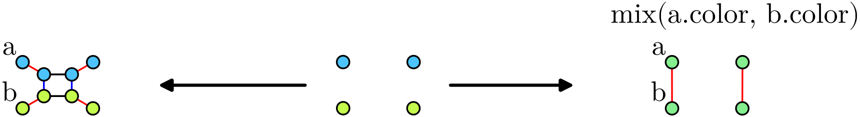




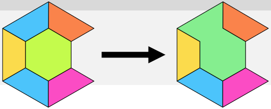
Breaking the geometric consistency



Constraint: nodes in a $\langle 0, 1 \rangle$-orbit should have the same color.



Breaking the geometric consistency



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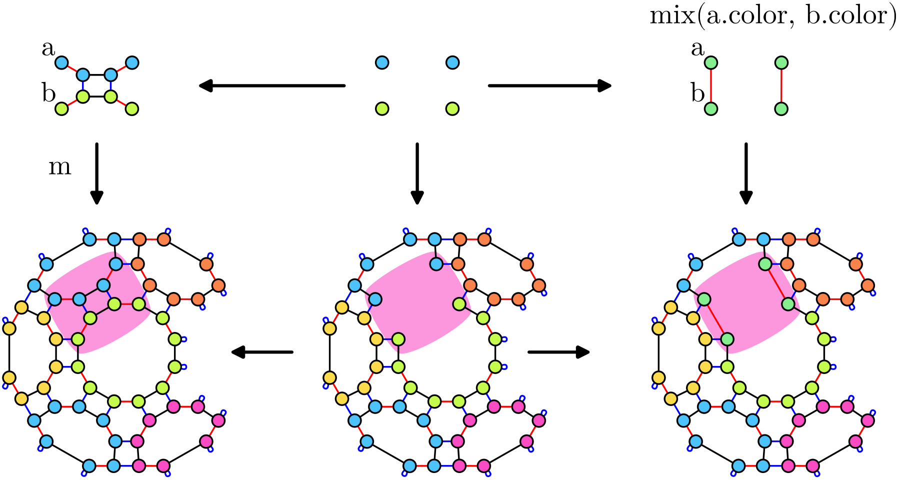

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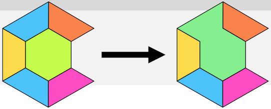

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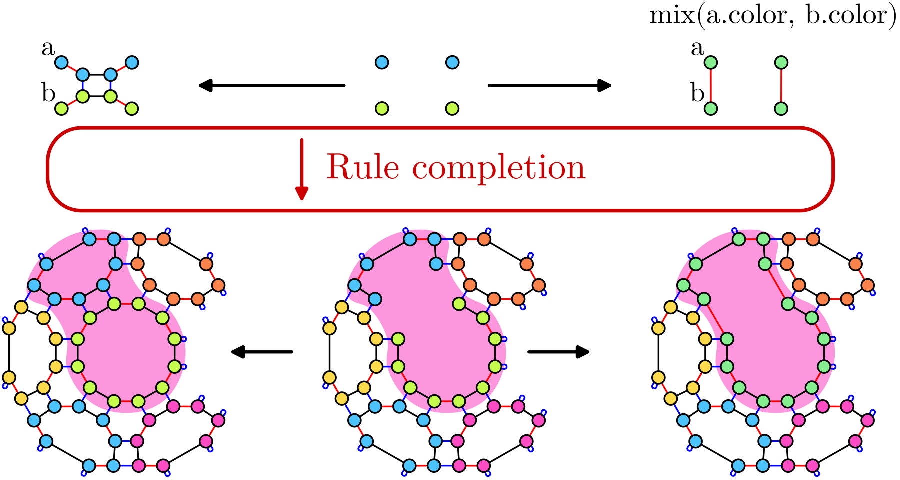
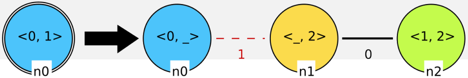


Main results



- Expressing Jerboa's DSL[1] with categorical constructions:
  - Graph products (topology)
  - Rule completion (geometry)
- Weaker consistency conditions:
  - Necessary and sufficient conditions on DPO rules
  - Reduce false negatives in the analyzer (safeguard for inference)
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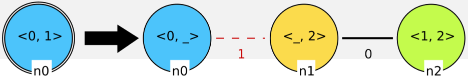

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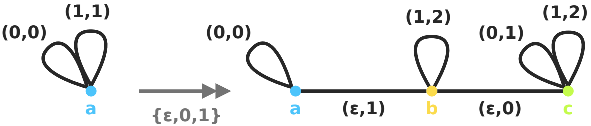

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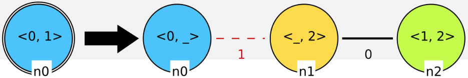

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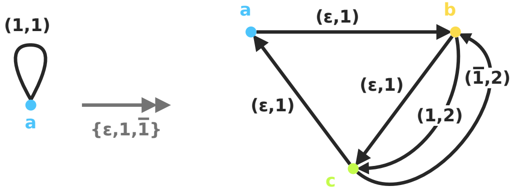

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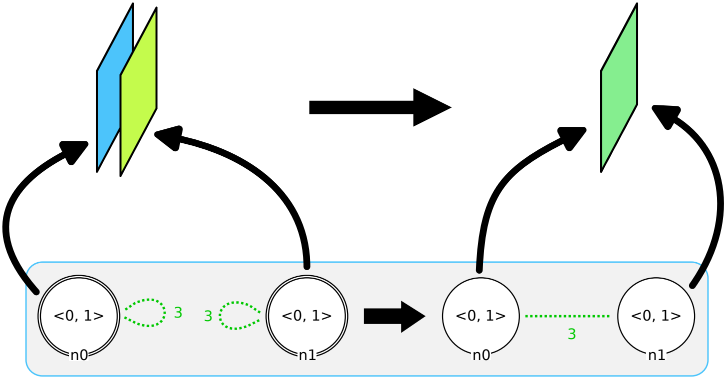
Inferring geometric modeling operations

- Retrieving the operation described by an example.

Reversing the instantiation process



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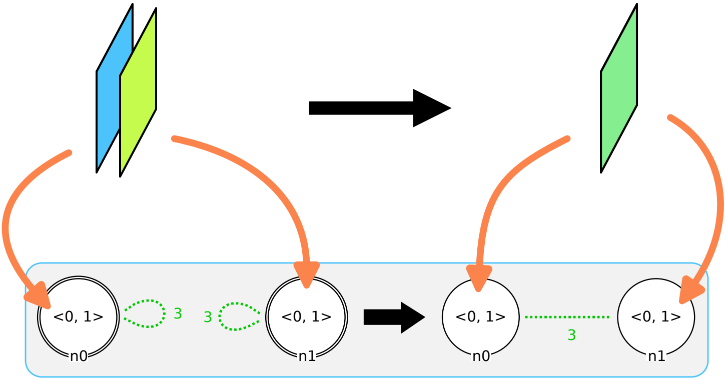
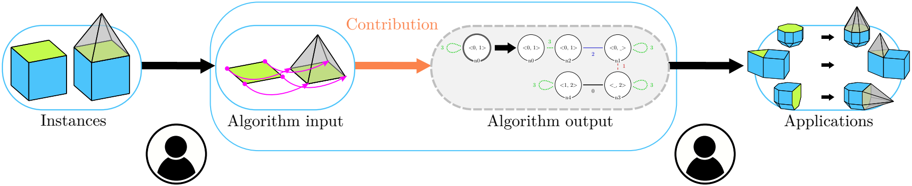


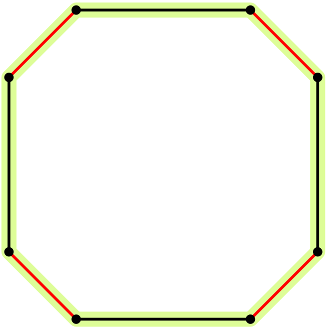
Inference workflow



▶ Input: A graph G encoding the preservation relation between two partial Gmaps, an orbit type $\langle o \rangle$ and a dart a of G.

▶ Output: A graph $\mathcal{S}$ that encodes the Jerboa rule with the variable $\langle o \rangle$, given that the operation is applied to the dart a.

Folding a joint representation of the rule



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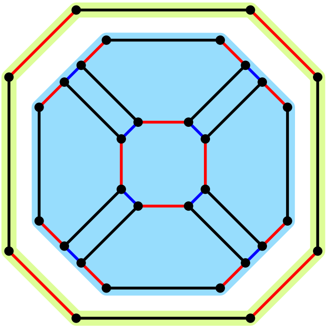

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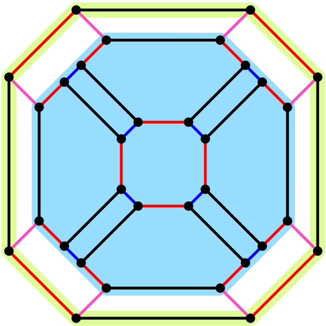

Color legend: 0, 1, 2, κ.

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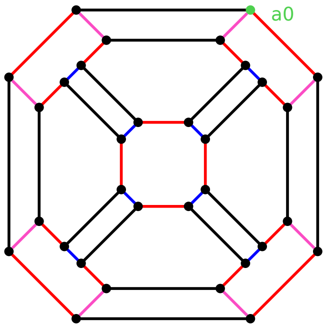

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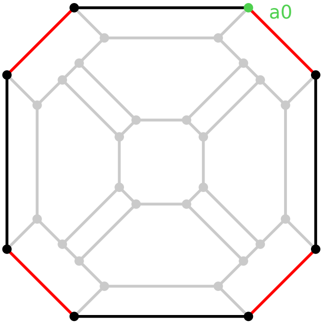
Besides the two Gmaps and the preservation links, we chose a dart in the initial Gmap and an orbit type.

▶ Graph traversal algorithm.
Iteratively applying two foldings:

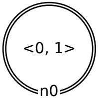
- Folding of a node.
- Folding of the arcs.

▶ Illustration on face triangulation with the orbit type $\langle 0, 1 \rangle$ and the dart $a0$.

Execution



Creation of the hook (orbit $\langle 0, 1 \rangle$).



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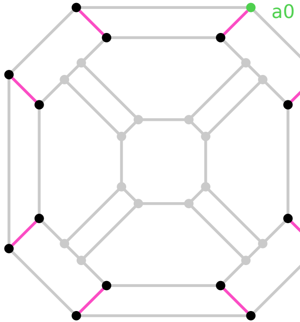
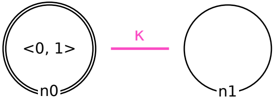


Folding of the arcs.



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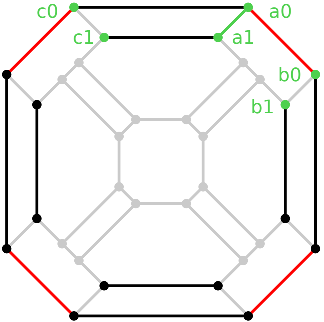
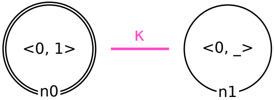


Folding of a node.



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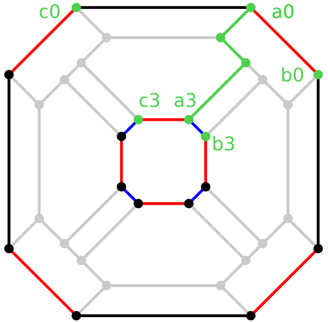

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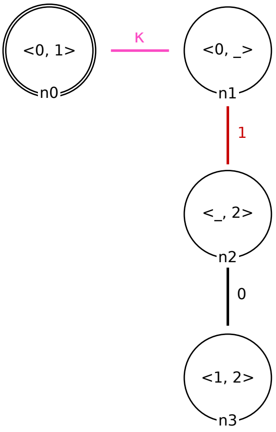
The algorithm terminates.



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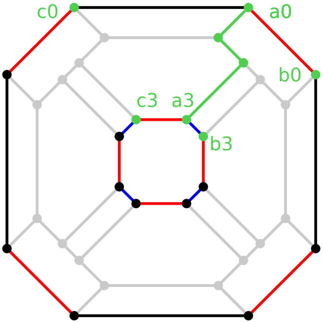

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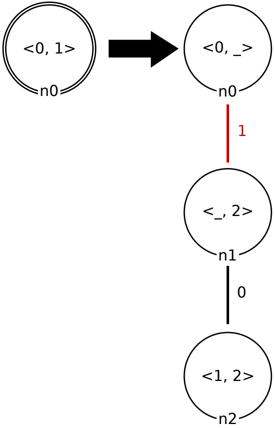
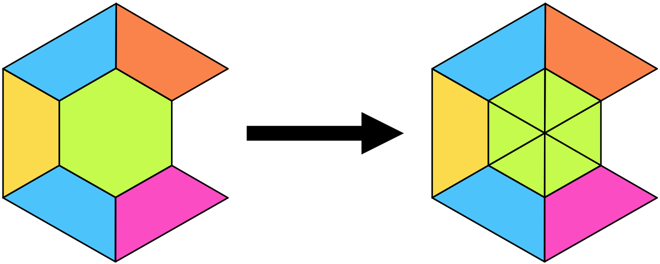
Splitting the joint representation.



Results

- Correctness: The algorithm returns a topological folding of the rule if it exists and halts otherwise.

- What about cases where we cannot fold the rule? Example with the orbit $\langle 0, 1, 2 \rangle$.



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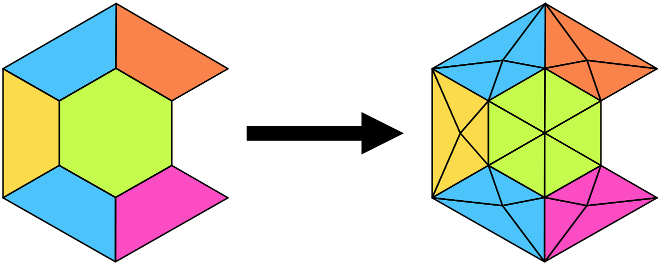

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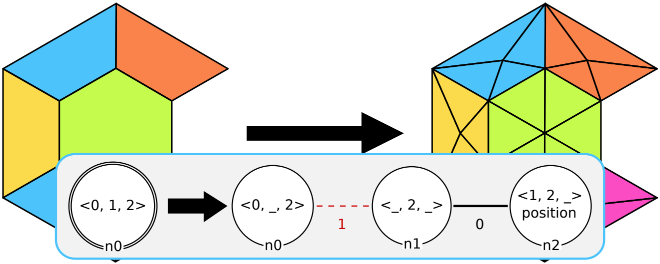
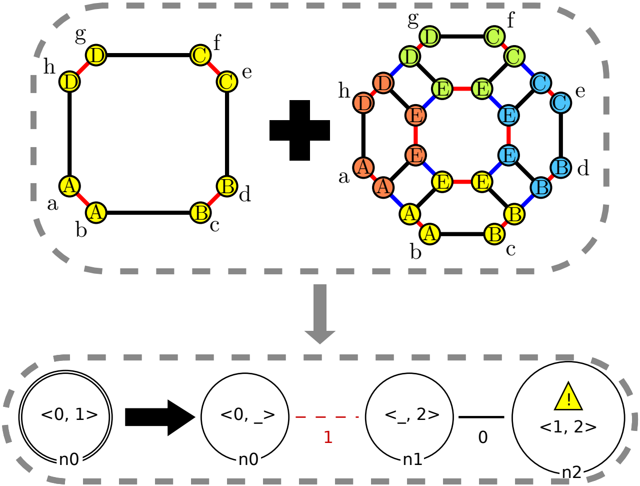


Objective



The rule is missing its embedding expressions.

Method (inference of positions)

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$$p = \sum_{i=0}^{k} w_i p_i + t$$

where:

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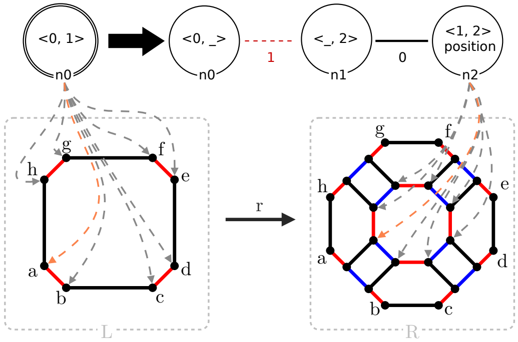

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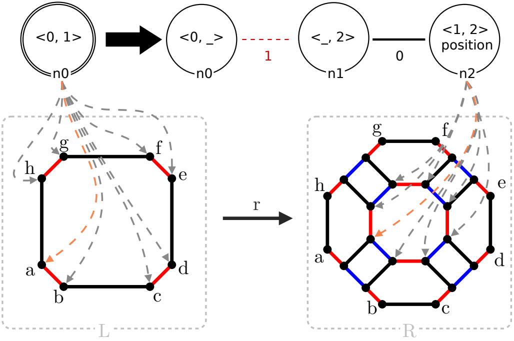

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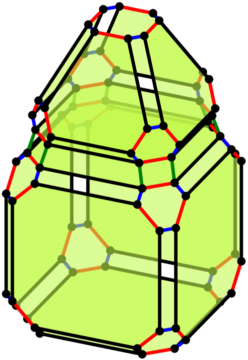
Solution: Exploit the topology.

- Use points of interest that share the same expression.

Points of interest



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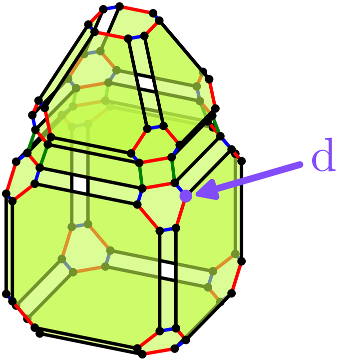
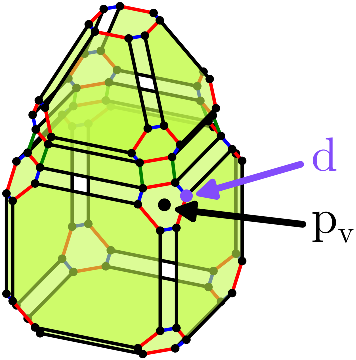


Points of interest

with

- p_v : vertex

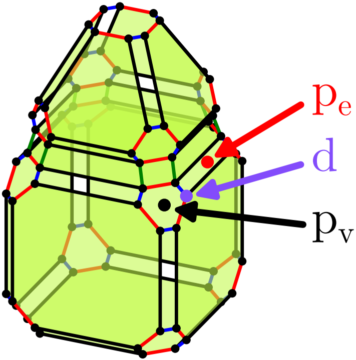


$$p_v = middle(position_{\langle 1,2,3 \rangle}(d))$$

Points of interest

with

- p_v : vertex
- p_e : edge midpoint

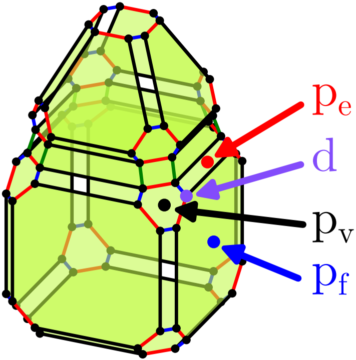


$$p_e = middle(position_{\langle 0,2,3 \rangle}(d))$$

Points of interest

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter



$$p_f = middle(position_{\langle 0,1,3 \rangle}(d))$$

Points of interest

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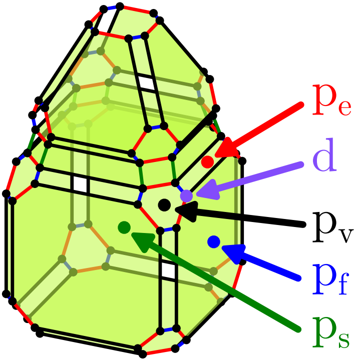
- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter
- p_s : volume barycenter



$$p_s = middle(position_{\langle 0,1,2\rangle}(d))$$

Points of interest

with

- p_v : vertex
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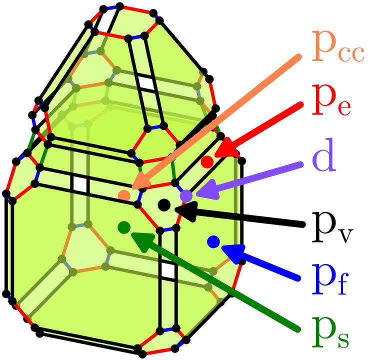
- p_f : face barycenter
- p_s : volume barycenter
- p_{cc} : CC barycenter



$$p_{cc} = middle(position_{\langle 0,1,2,3 \rangle}(d))$$

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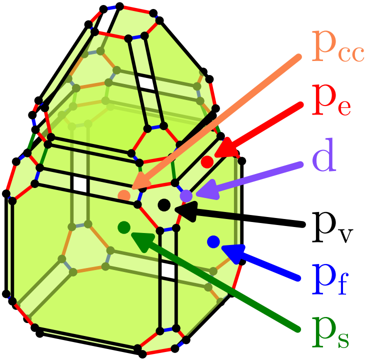
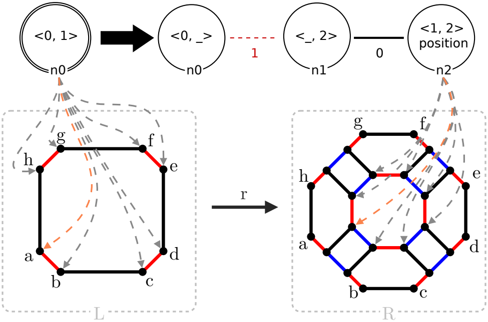


Thanks to the points of interest, the system is rewritten as:

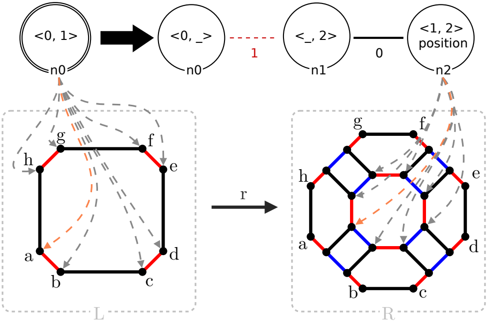
$$p = w_v p_v + w_e p_e + w_f p_f + w_s p_s + w_{cc} p_{cc} + t$$

Illustration



The position expression of $n2$ only depends on $n0$.

Illustration



The position expression of $n2$ only depends on $n0$.

$$n2.position = \underbrace{w_v\, n0.p_v}_{vertex} + \underbrace{w_e\, n0.p_e}_{edge} + \underbrace{w_f\, n0.p_f}_{face} + \underbrace{w_s\, n0.p_s}_{volume} + \underbrace{w_{cc}\, n0.p_{cc}}_{cc} + t$$

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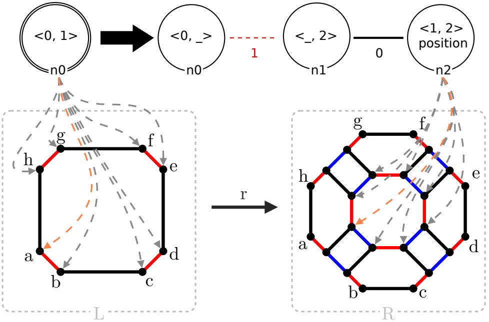

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- One equation per dart (8 darts).

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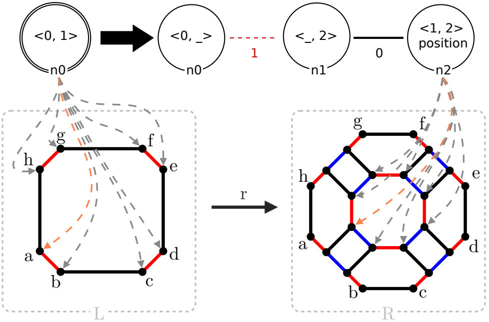

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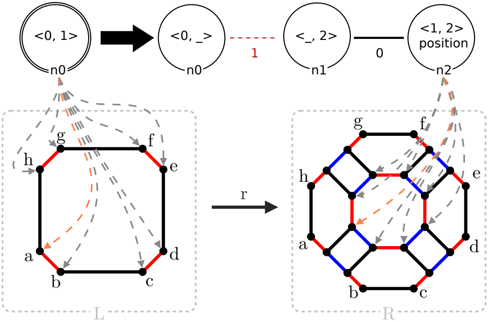

The position expression of $n2$ only depends on $n0$.

- One equation per dart (8 darts).
- Split per coordinate (on x, y, z).
- 24 equations and 8 variables.

$$n2.position = \underbrace{w_v\, n0.p_v}_{vertex} + \underbrace{w_e\, n0.p_e}_{edge} + \underbrace{w_f\, n0.p_f}_{face} + \underbrace{w_s\, n0.p_s}_{volume} + \underbrace{w_{cc}\, n0.p_{cc}}_{cc} + t$$

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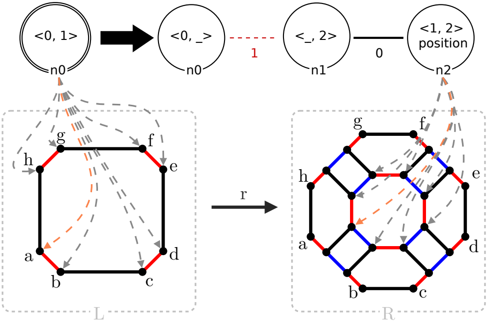

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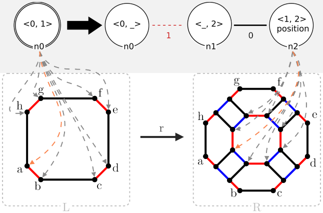
- Solved as a CSP. Solvers used: OR-Tools (Google), Z3 (Microsoft)

Solving the barycentric triangulation



- Global equation:

$$n2.position = w_v\, n0.p_v + w_e\, n0.p_e + w_f\, n0.p_f + w_s\, n0.p_s + w_{cc}\, n0.p_{cc} + t$$

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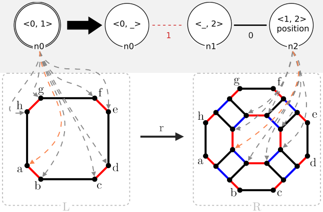


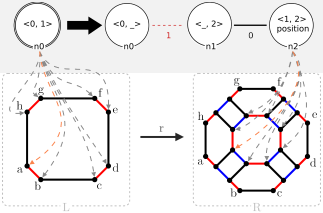
- Global equation:

$$n2.position = w_v\, n0.p_v + w_e\, {\color{red} n0.p_e} + w_f\, {\color{blue} n0.p_f} + w_s\, {\color{green} n0.p_s} + w_{cc}\, {\color{orange} n0.p_{cc}} + t$$

- Generated system (only on x and y)

$$\begin{cases} (0.5;0.5) = w_v * (0;0) + w_e * {\color{red}(0.5;0)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\ (0.5;0.5) = w_v * (1;0) + w_e * {\color{red}(0.5;0)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\ (0.5;0.5) = w_v * (1;0) + w_e * {\color{red}(1;0.5)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\ (0.5;0.5) = w_v * (1;1) + w_e * {\color{red}(1;0.5)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{cases}$$

Solving the barycentric triangulation



- Global equation:

$$n2.position = w_v\, n0.p_v + w_e\, n0.p_e + w_f\, n0.p_f + w_s\, n0.p_s + w_{cc}\, n0.p_{cc} + t$$

- Generated system (only on x and y)

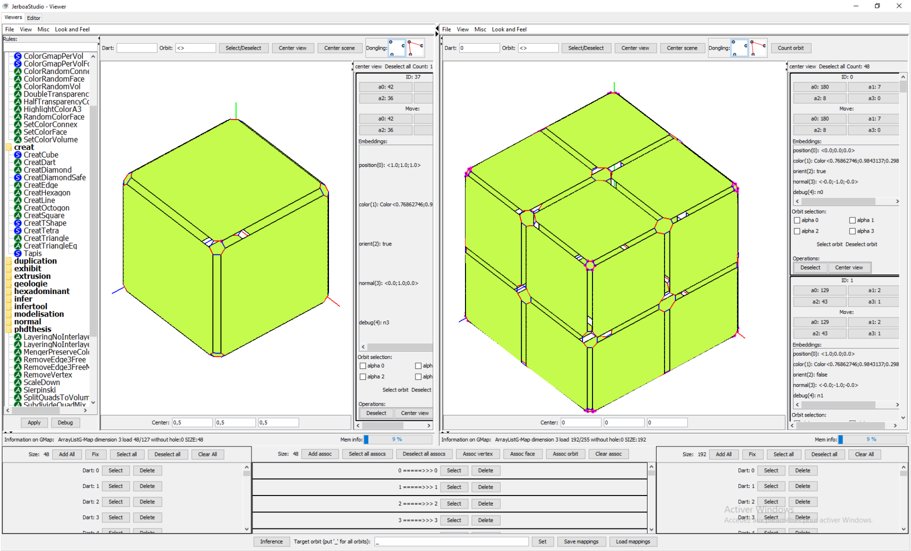
$$\begin{cases} (0.5;0.5) = w_v * (0;0) + w_e * (0.5;0) + w_f * (0.5;0.5) + w_s * (0.5;0.5) + w_{cc} * (0.5;0.5) + (tx;ty) \\ (0.5;0.5) = w_v * (1;0) + w_e * (0.5;0) + w_f * (0.5;0.5) + w_s * (0.5;0.5) + w_{cc} * (0.5;0.5) + (tx;ty) \\ (0.5;0.5) = w_v * (1;0) + w_e * (1;0.5) + w_f * (0.5;0.5) + w_s * (0.5;0.5) + w_{cc} * (0.5;0.5) + (tx;ty) \\ (0.5;0.5) = w_v * (1;1) + w_e * (1;0.5) + w_f * (0.5;0.5) + w_s * (0.5;0.5) + w_{cc} * (0.5;0.5) + (tx;ty) \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{cases}$$

- Solution found:
  - $w_v = 0.0$
  - $w_e = 0.0$
  - $w_f = 1.0$
  - $w_s = 0.0$
  - $w_{cc} = 0.0$
  - $t = (0.0, 0.0)$

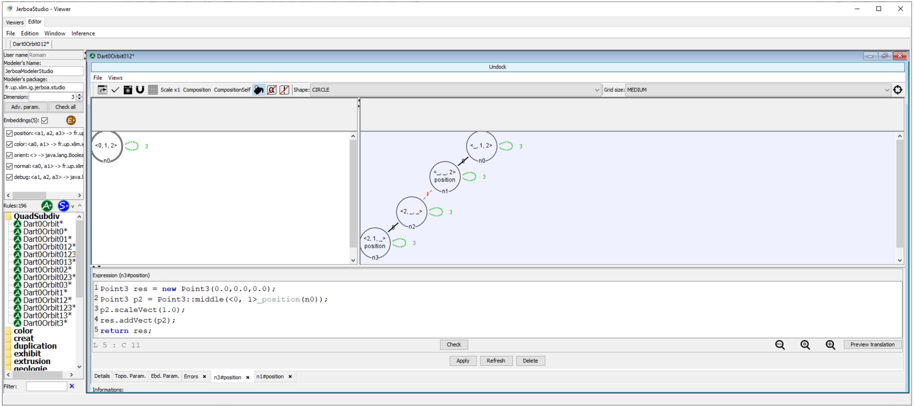
JerboaStudio and applications

- Implementation of the inference mechanism in Jerboa.

JerboaStudio: inferring the quad subdivision



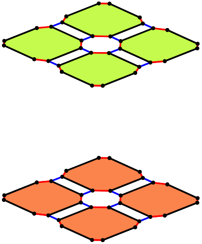
Folding the quad subdivision



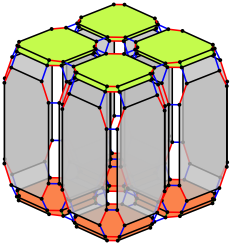
- 768 possible schemes
- 48 schemes tried (marking).
- 14 schemes built (removal of isomorphic rules).

Example inspired from geology

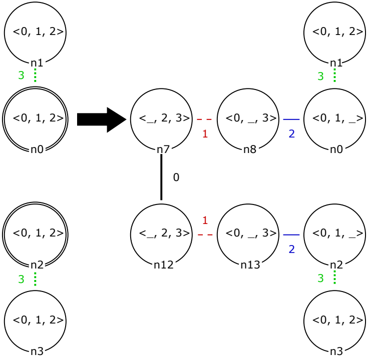
Before



After



Operation



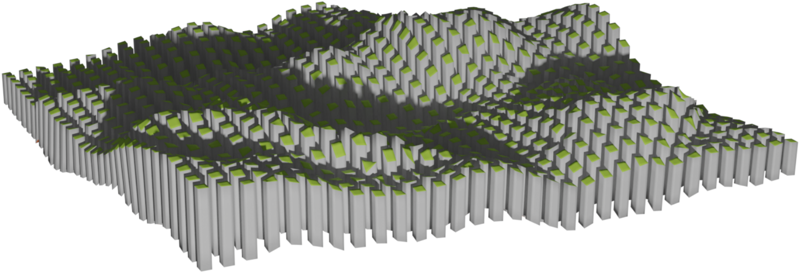
Inference time: ~ 3 ms

Example inspired from geology

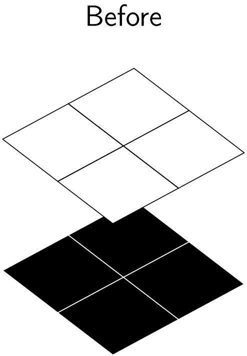
Before

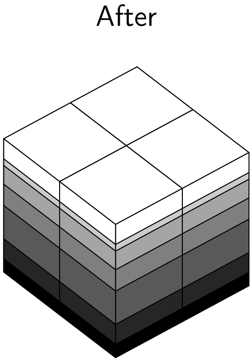
Example inspired from geology

After



Example inspired from geology (part 2)

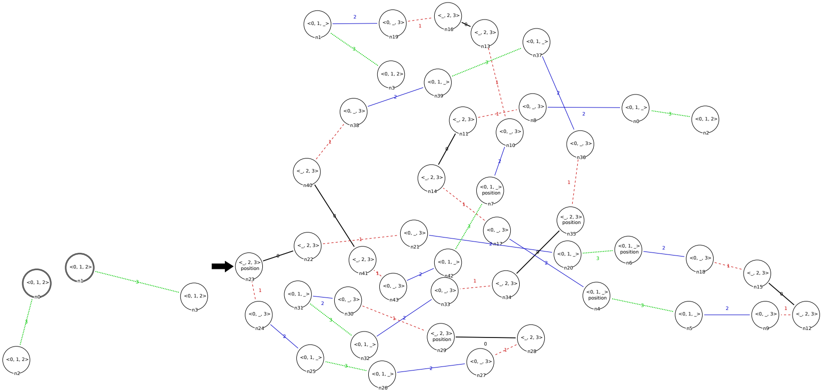




- We infer interpolations both for the positions and the colors.

Example inspired from geology (part 2)

Operation



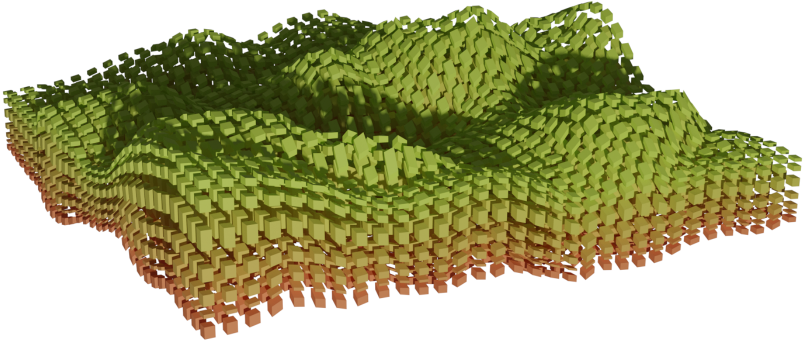
Inference time: $\sim$ 26 ms for the topology,
$\sim$ 549 ms for the embedding expressions

Example inspired from geology (part 2)

Before

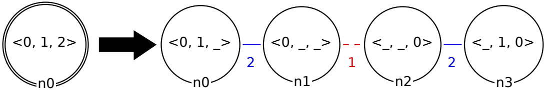
Example inspired from geology (part 2)

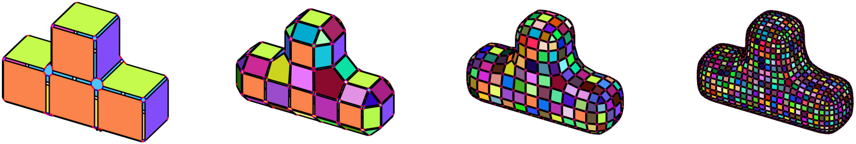
After



Doo-Sabin subdivision[1]

- Rule scheme used and inferred:

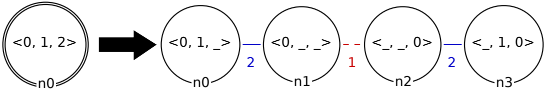


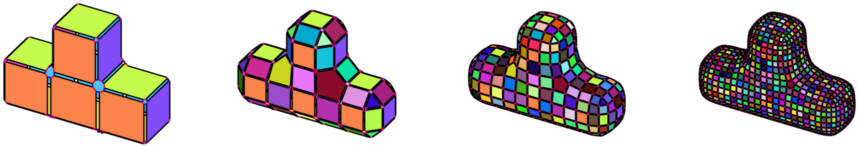


[1]Doo et al. 1978.

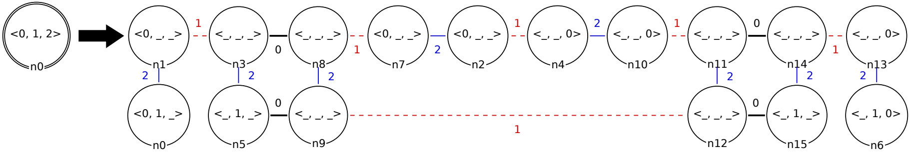
Doo-Sabin subdivision[1]

- Rule scheme used and inferred:





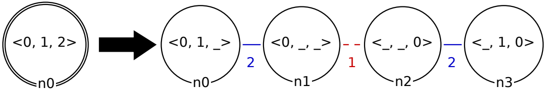
- *2nd* iteration:

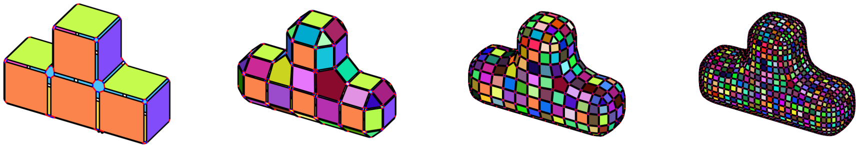


[1] Doo et al. 1978.

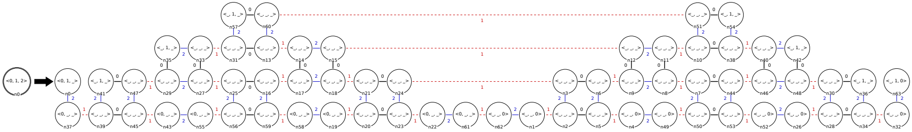
Doo-Sabin subdivision[1]

- Rule scheme used and inferred:



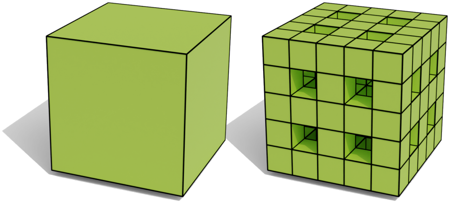


- 3rd iteration:



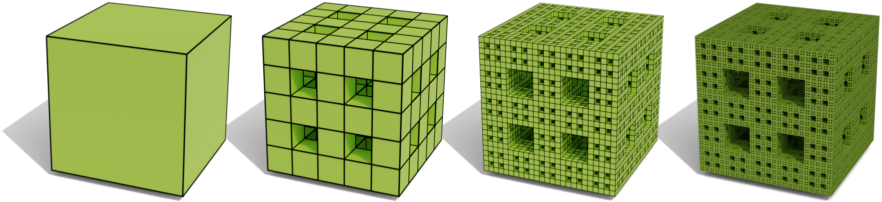
[1] Doo et al. 1978.

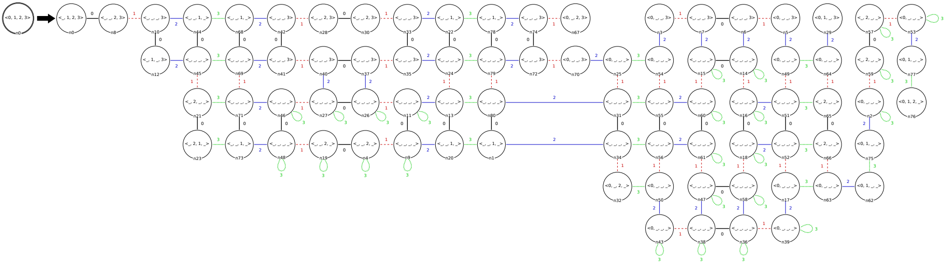
Menger $(2, 2, 2)$[1]



[1] Richaume et al. 2019.

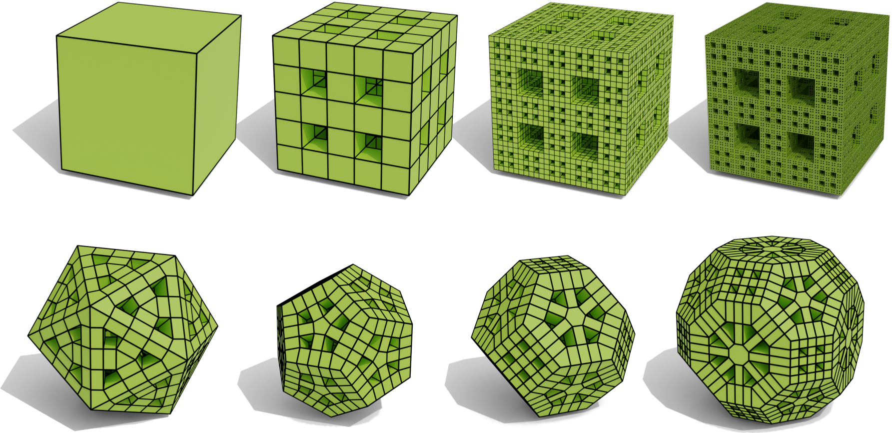
Menger $(2,2,2)$[1]





[1] Richaume et al. 2019.

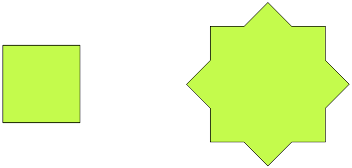
Menger $(2, 2, 2)$[1]



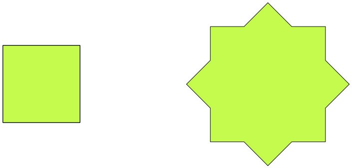
[1] Richaume et al. 2019.

Edge cases

- Von Koch's snowflake generated with L-systems

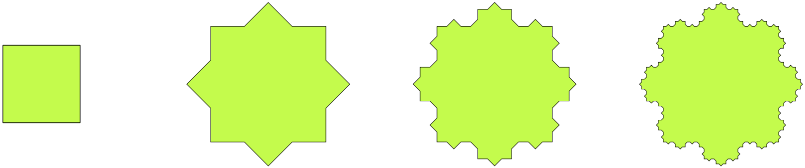


- Inferred:

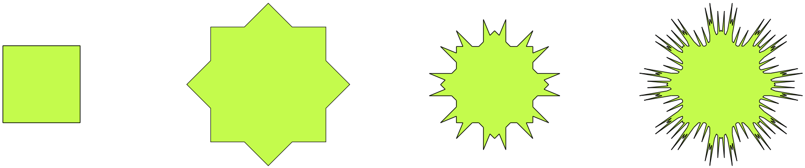


Edge cases

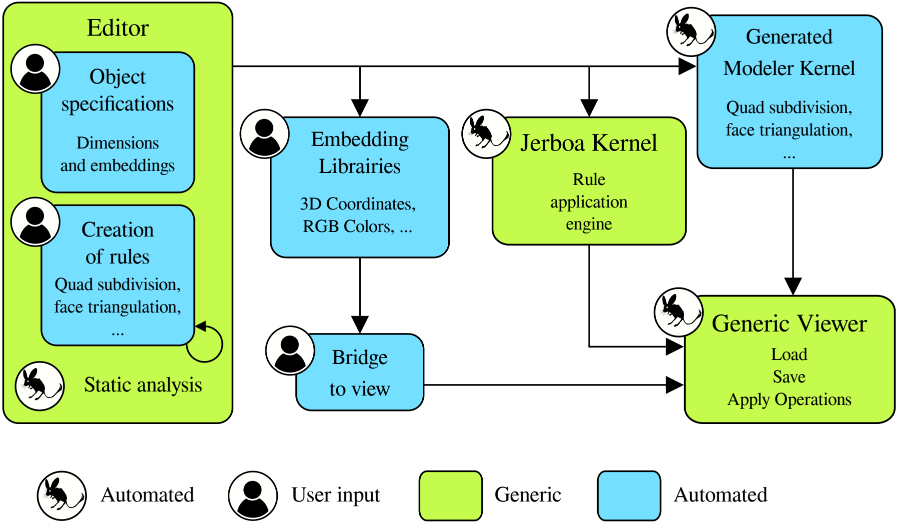
- Von Koch's snowflake generated with L-systems



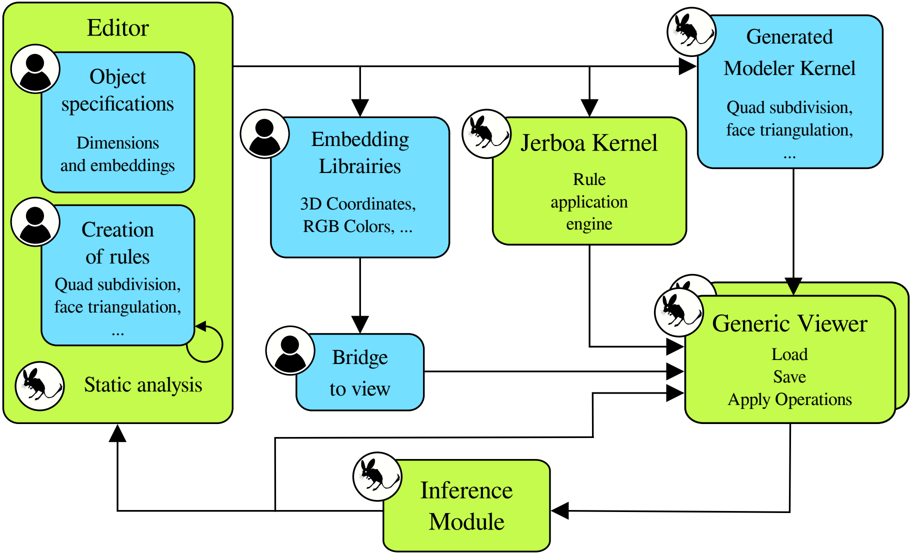
- Inferred:



JerboaStudio's architecture



JerboaStudio's architecture

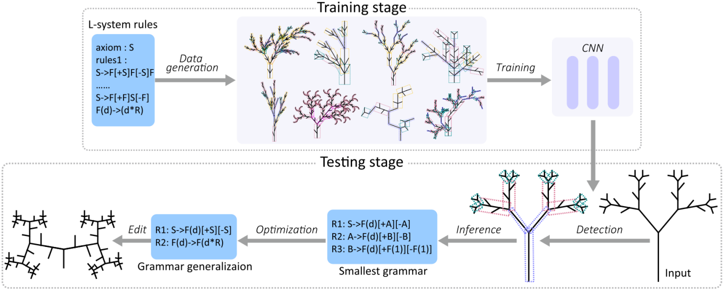


Conclusion

- Related works, main contributions, and future works.

Other lines of research on inference

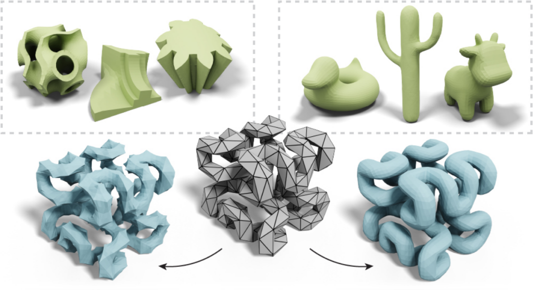
- Inferring the generation of an object:
  - Inverse procedural modeling: retrieving parameters.[1]
  - L-systems: retrieving formal rules.[2] Illustration from (Guo et al. 2020).
  - Constructive solid geometry: retrieving sequences of operations.[3]



[1] Wu et al. 2014; Emilien et al. 2015.
[2] Santos et al. 2009; Šťava et al. 2010.
[3] Sharma et al. 2018; Kania et al. 2020; Xu et al. 2021.

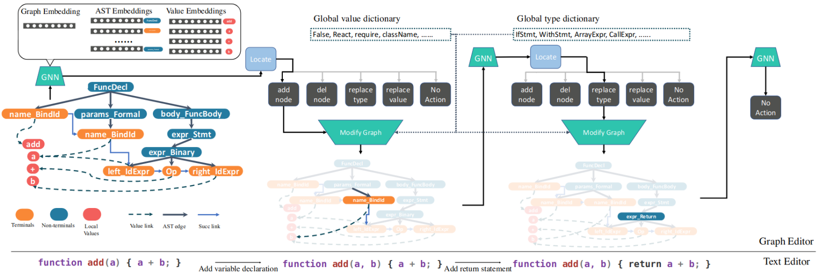
Other lines of research on inference

- Inferring the generation of an object
- Pure geometry
  - Retrieve non-linear weights of a Loop-based subdivision scheme for mesh refinement. Illustration from (Liu et al. 2020).



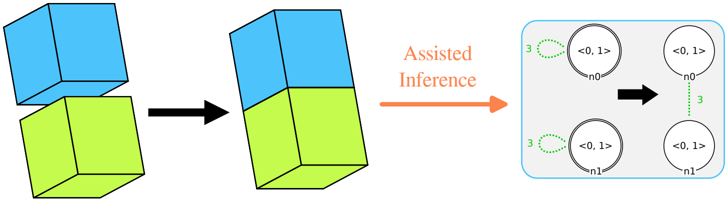
Other lines of research on inference

- Inferring the generation of an object
- Pure geometry
- Graph transformations
  - Domain-based inference mechanism retrieving or exploiting graph transformations.[1] Illustration from (Dinella et al. 2020).



[1] Alshanqiti et al. 2016; López-Fernández et al. 2019.

Main contributions



Inference of modeling operations:

- Topological folding algorithm
- Values of interest and CSP

▶ JerboaStudio.

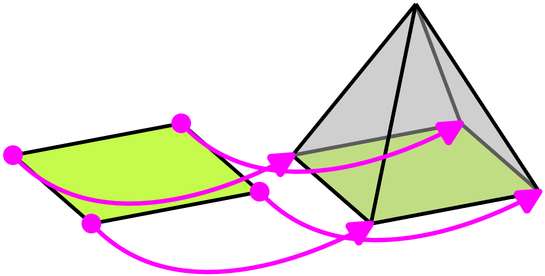
Graph transformations for geometric modeling:

- Graph products
- Rule completion

▶ Unified framework to study generalized and oriented maps.

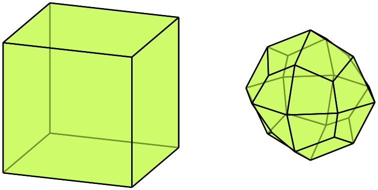
Future works

- Automatic mapping
  - Cumbersome step in the inference workflow.



Future works

- Automatic mapping
- Other hypotheses for the geometric inference
  - Most subdivision schemes rely on other computations: the Catmull-Clark subdivision.[1]



```
// From Catmull and Clark 1978, Recursively generated
// B-spline surfaces on arbitrary topological meshes

// n1#position
// midpoint of the incident face
Point3 face1Mid = Point3::middle(<0,1>_position(n0));
// midpoint of the adjacent face
Point3 face2Mid = Point3::middle(<0,1>_position(n0@2));
// average of the face points
Point3 faceMid = Point3::middle(face1Mid, face2Mid);
// midpoint of the edge
Point3 edgeMid = Point3::middle(<0>_position(n0));
// average of the edge and face points
return Point3::middle(faceMid, edgeMid);
```

Out of scope

[1]Catmull et al. 1978.

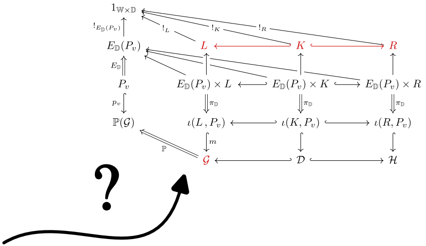
Future works

- Automatic mapping
- Other hypotheses for the geometric inference
- Inference in graph transformations
  - Formalize the inference mechanism with categorical constructions.

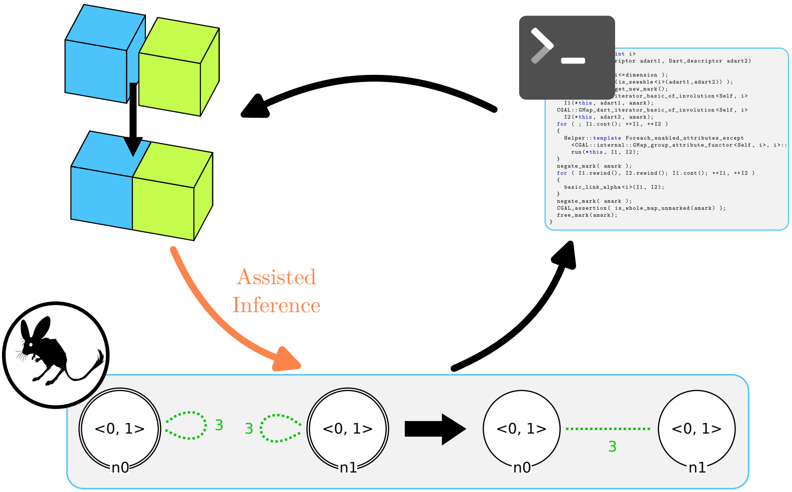
Algorithm: Topological folding algorithm

Input: A Gmap G, an orbit type $\langle o \rangle$, and a dart a of G.
Output: The scheme graph $\mathcal{S}$ such that the instantiation $\iota^{\langle o \rangle}(\mathcal{S}, G\langle o \rangle(a))$ maps (h, a) onto a.

```
Q ← empty queue
// Empty scheme graph
S ← ∅
// Construction of the hook (Step 1)
h ← createHook(S,⟨o⟩)
enqueue(Q,h)
// Traversal (Step 2)
while Q not empty do
    m ← dequeue(Q)
    foreach d ∈ (0..n)\ orbType(m) do
        // Extension of the explicit arcs incident to v (Step 2.1)
        v ← arcExt(G,m,d)
        // Construction of the orbit type decorating v (Step 2.2)
        buildOrbType(G,v)
        enqueue(Q,v)
return S
```



Thank you for listening



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