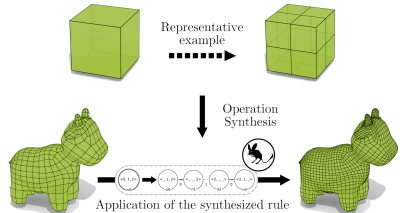


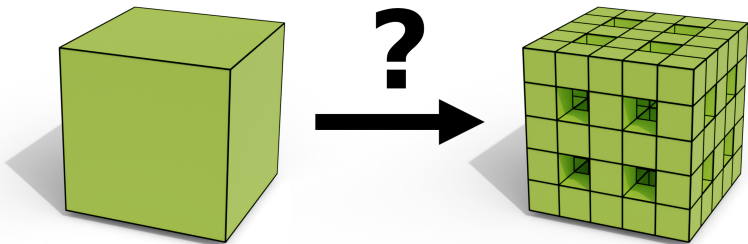
Program Synthesis for Geometric Modeling

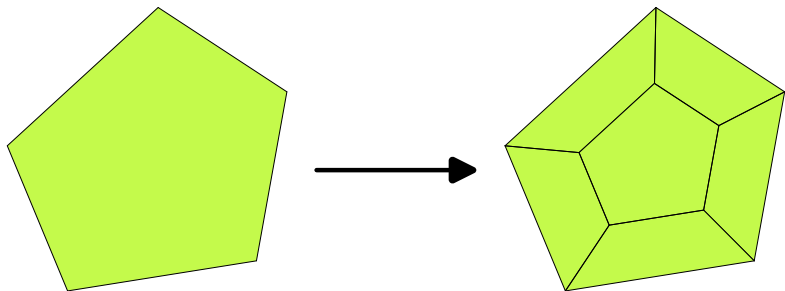
Romain Pascual, Pascale Le Gall,
Hakim Belhaouari, and
Agnès Arnould

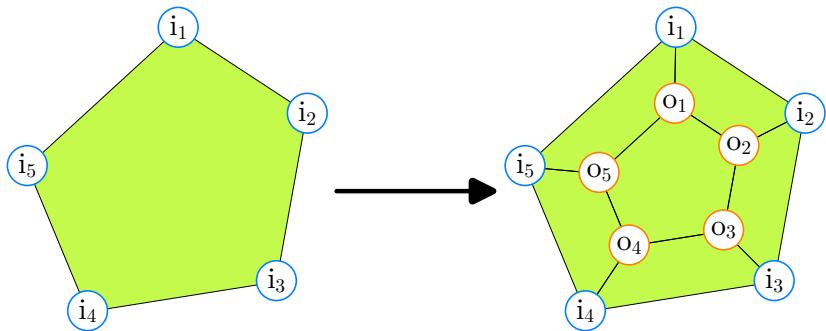
September 09, 2025

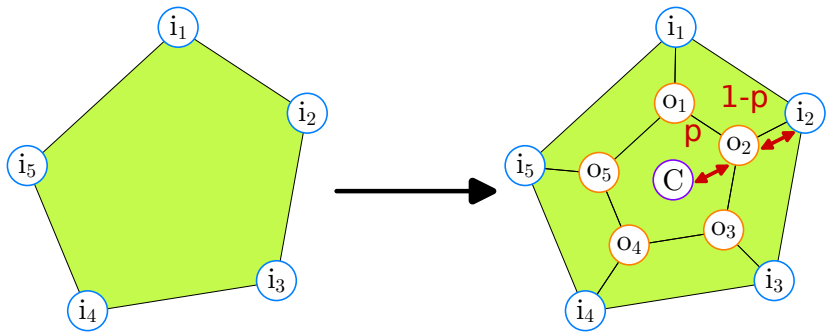
LOPSTR 2025



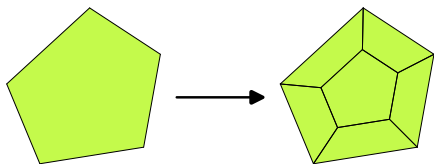




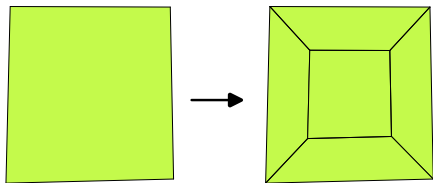




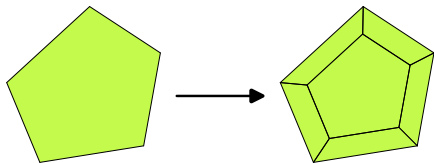
$$o_k = p \cdot i_k + (1 - p) \cdot \frac{1}{5} \sum_{n=1}^5 i_n$$



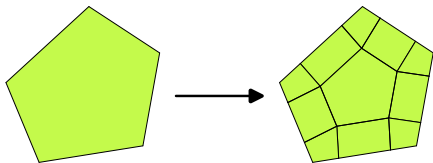
Input-to-output example



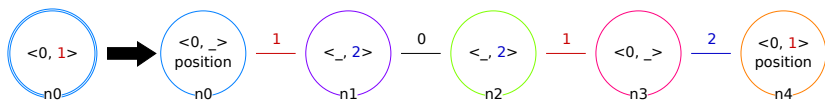
Different input topology



Different output geometry



Different operation



Position for n_0

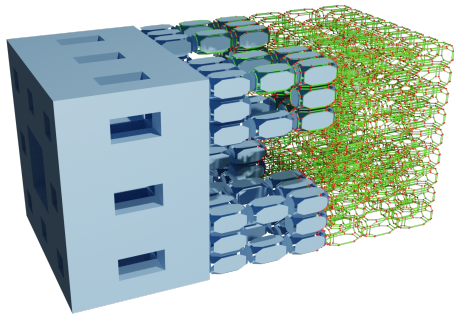
```
return n0.position;
```

Position for n_4

```
return Point3::midpt(  
    Point3::middle(<0,1> _position(n0)),  
    n0.position);
```

Embedded Generalized Maps

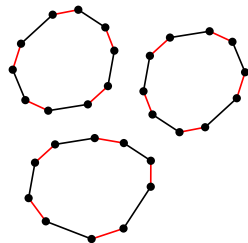
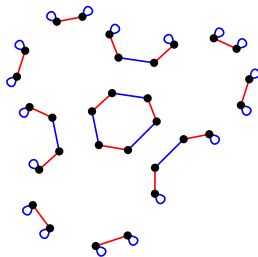
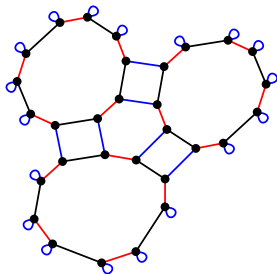
- How to represent objects?



Generalized Maps¹ (Topology)



Orbit: Sub-graph induced by a subset $\langle o \rangle$ of dimensions



Legend: 0, 1, 2

Vertices: orbits $\langle 1, 2 \rangle$

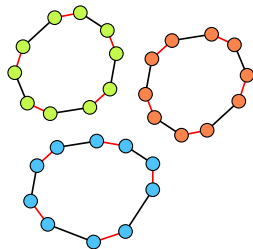
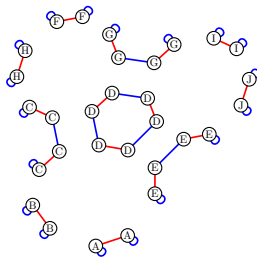
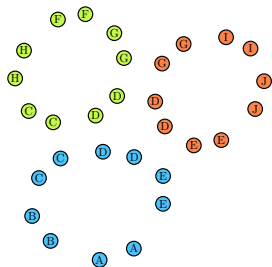
Faces: orbits $\langle 0, 1 \rangle$

¹Damiand et al. 2014.

Embeddings (Geometry)



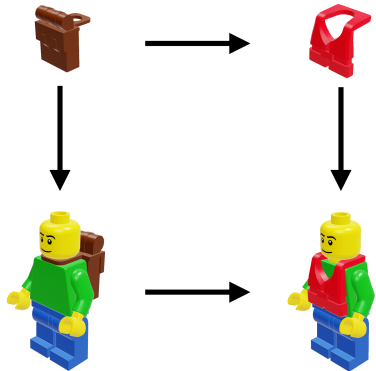
Embedding: function $\pi : \langle o_\pi \rangle \rightarrow \tau_\pi$
with τ_π an abstract data type



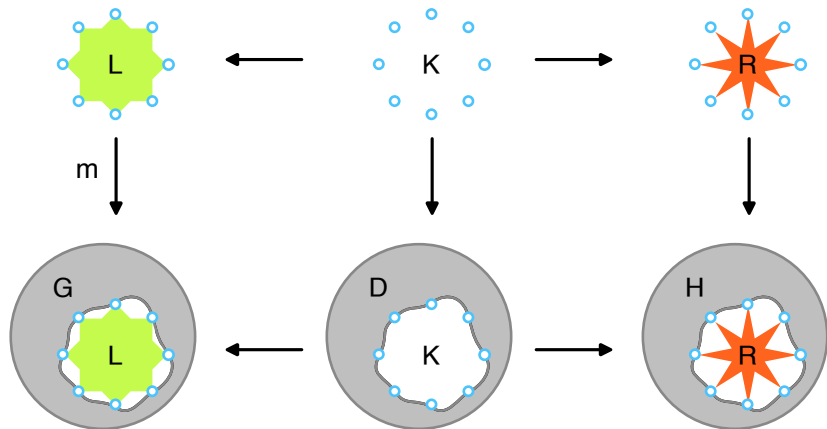
Legend: 0, 1, 2 position : $\langle 1, 2 \rangle \rightarrow \text{Point3}$ color : $\langle 0, 1 \rangle \rightarrow \text{ColorRGB}$

Gmap Rewriting

- How to modify objects?

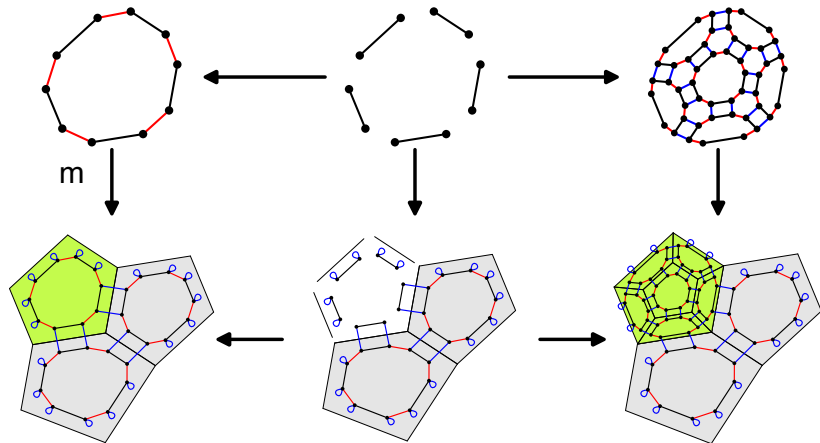
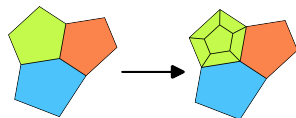


Graph Rewriting¹

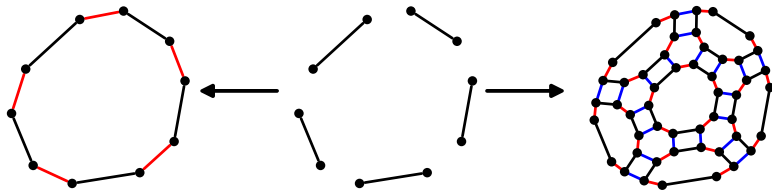
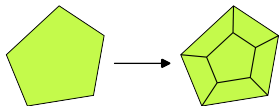


¹Rozenberg 1997; Ehrig et al. 2006; Heckel et al. 2020.

Gmap Rewriting

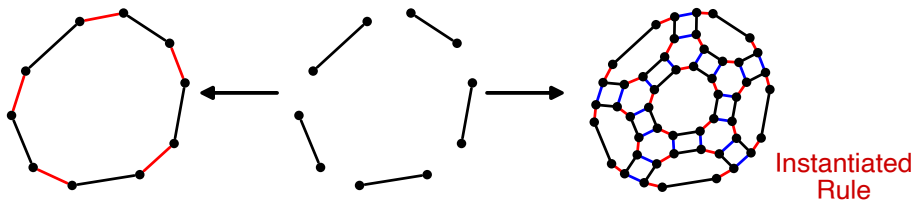
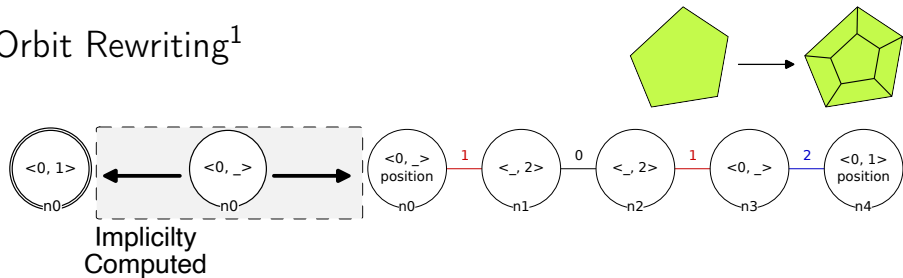


Orbit Rewriting¹



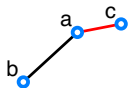
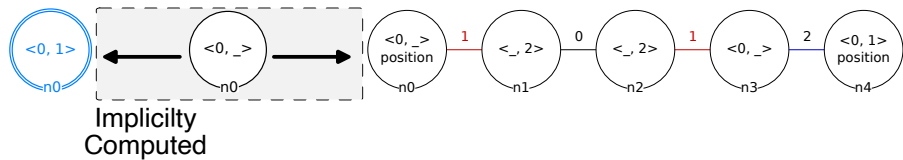
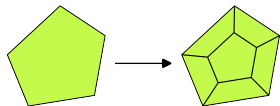
¹Pascual et al. 2022b.

Orbit Rewriting¹

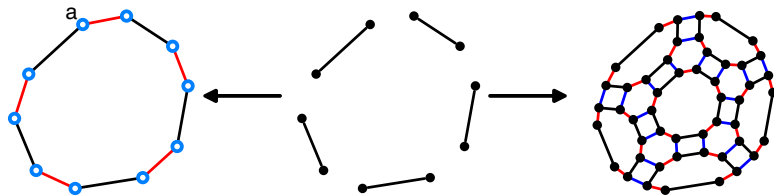


¹Pascual et al. 2022b.

Orbit Rewriting¹



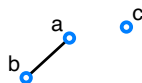
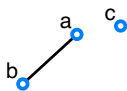
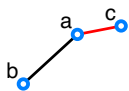
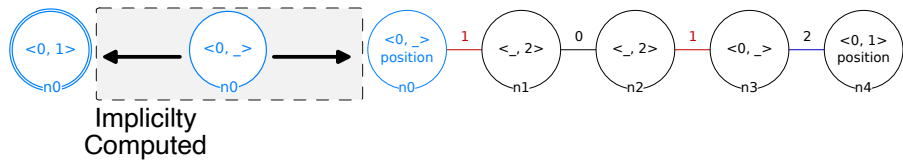
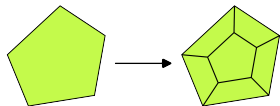
Local



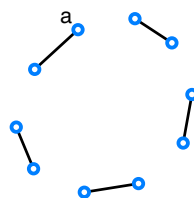
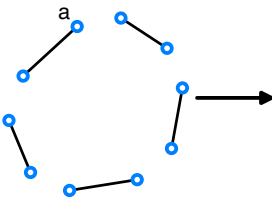
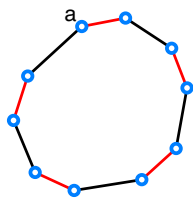
Instantiated Rule

¹Pascual et al. 2022b.

Orbit Rewriting¹



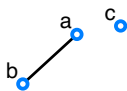
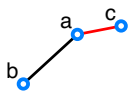
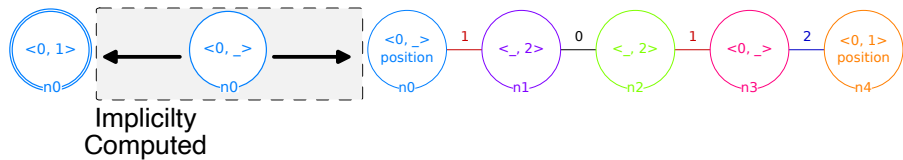
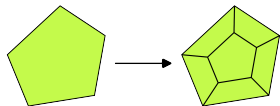
Local



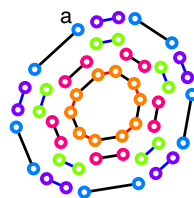
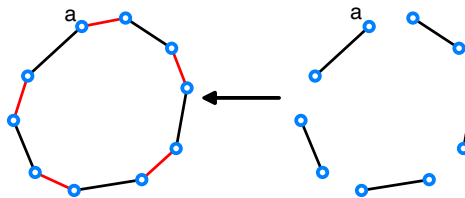
Instantiated Rule

¹Pascual et al. 2022b.

Orbit Rewriting¹



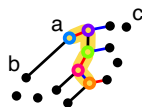
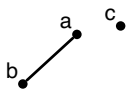
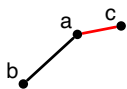
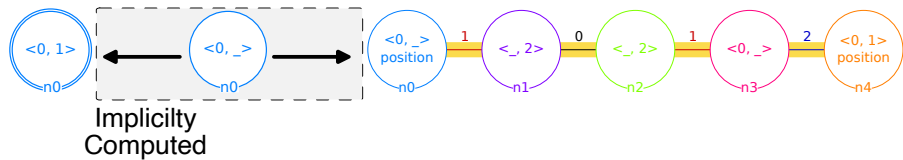
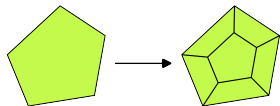
Local



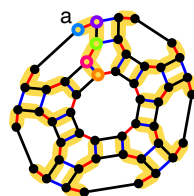
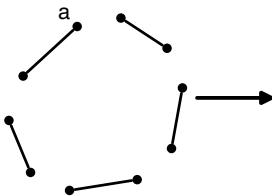
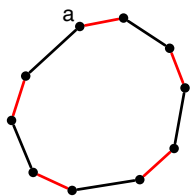
Instantiated Rule

¹Pascual et al. 2022b.

Orbit Rewriting¹



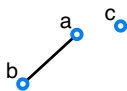
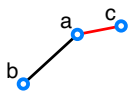
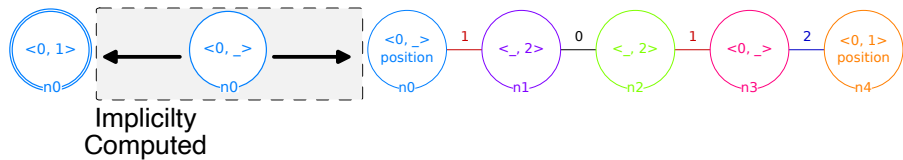
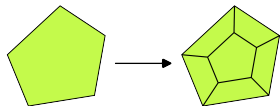
Local



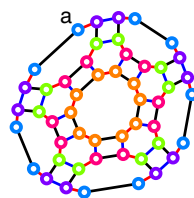
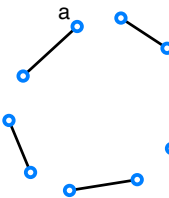
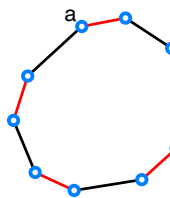
Instantiated Rule

¹Pascual et al. 2022b.

Orbit Rewriting¹



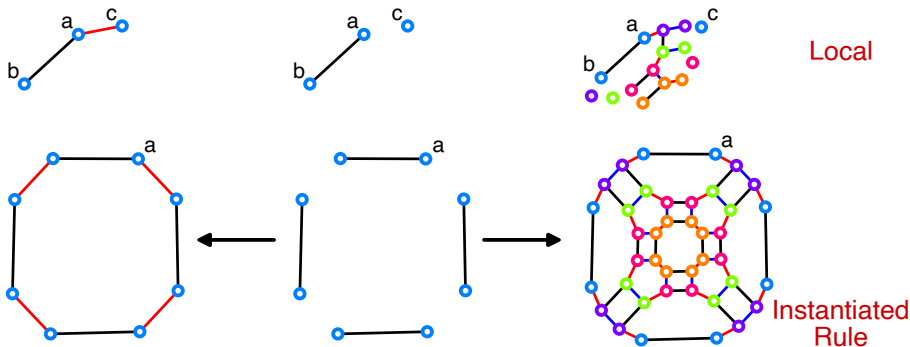
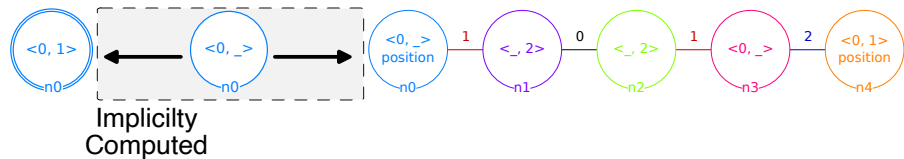
Local



Instantiated Rule

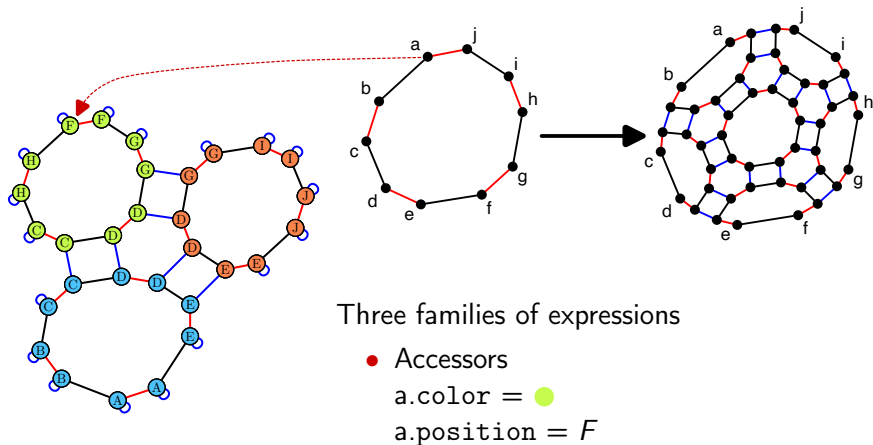
¹Pascual et al. 2022b.

Orbit Rewriting¹



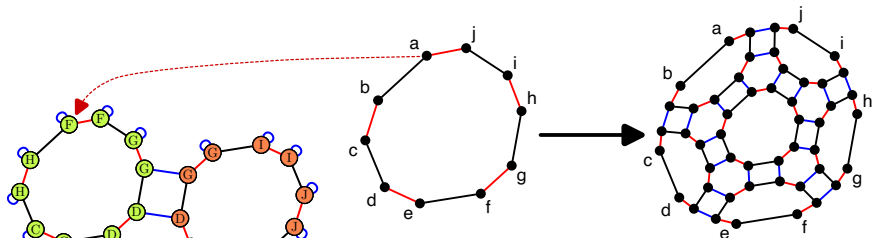
¹Pascual et al. 2022b.

Embedding Expressions¹



¹Bellet et al. 2017; Arnould et al. 2022.

Embedding Expressions¹



Three families of expressions

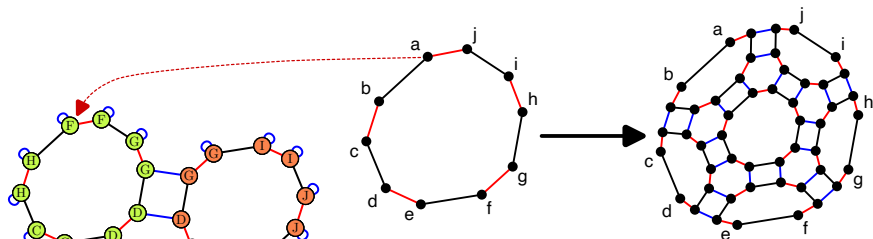
- Accessors

- Computations

$$\text{middle}(\{\bullet, \bullet\}) = \bullet$$

¹Bellet et al. 2017; Arnould et al. 2022.

Embedding Expressions¹



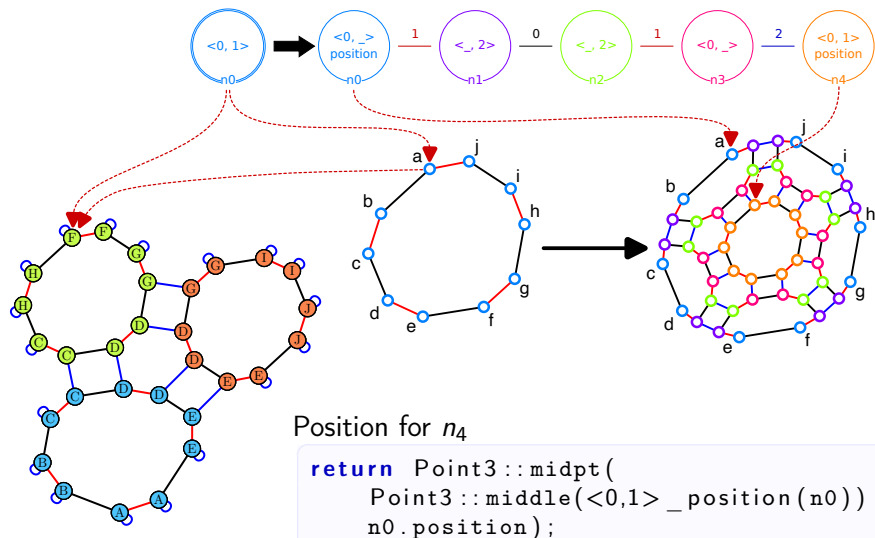
Three families of expressions

- Accessors
- Computations
- Gmap traversals

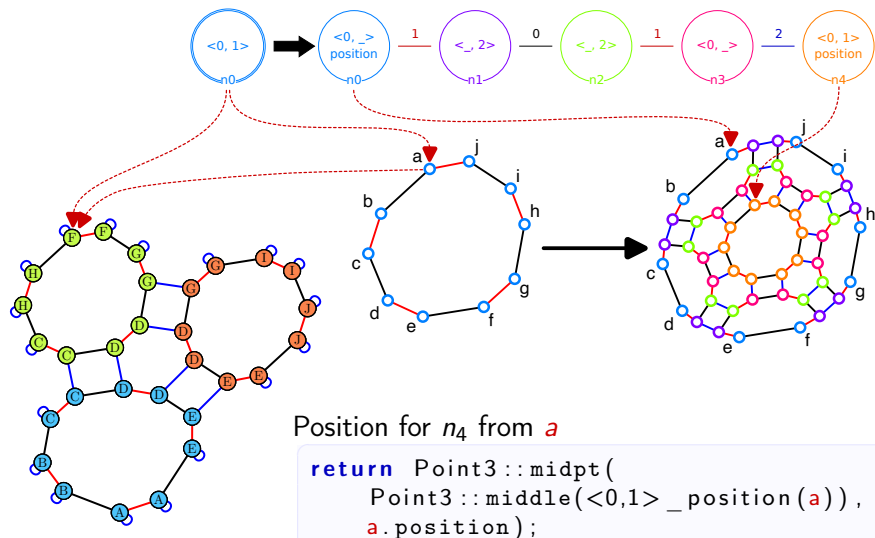
$$\text{position}_{\langle 0,1 \rangle}(a) = \{C, D, F, G, H\}$$

¹Bellet et al. 2017; Arnould et al. 2022.

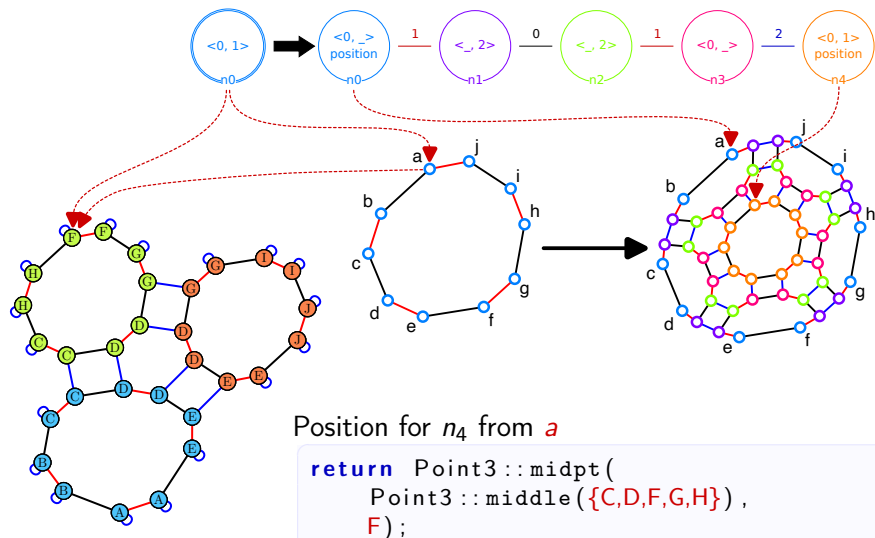
Extension to Rule Schemes



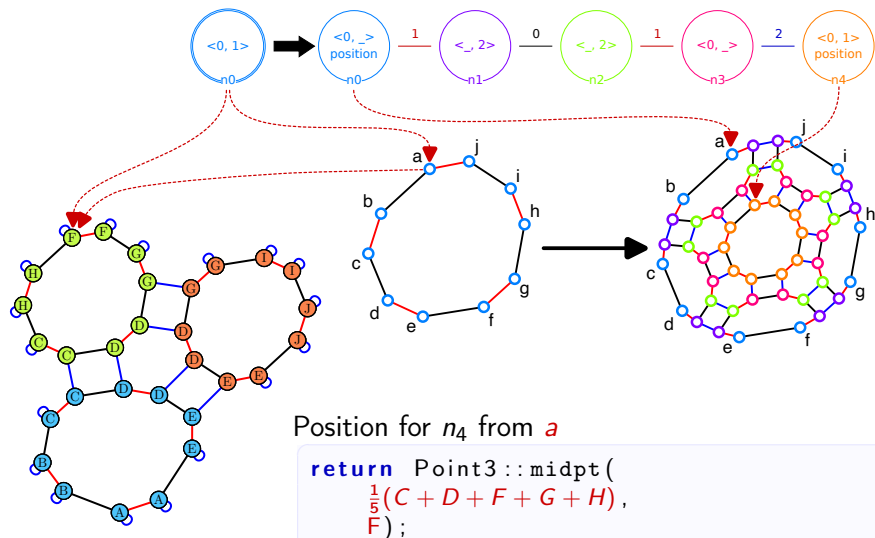
Extension to Rule Schemes



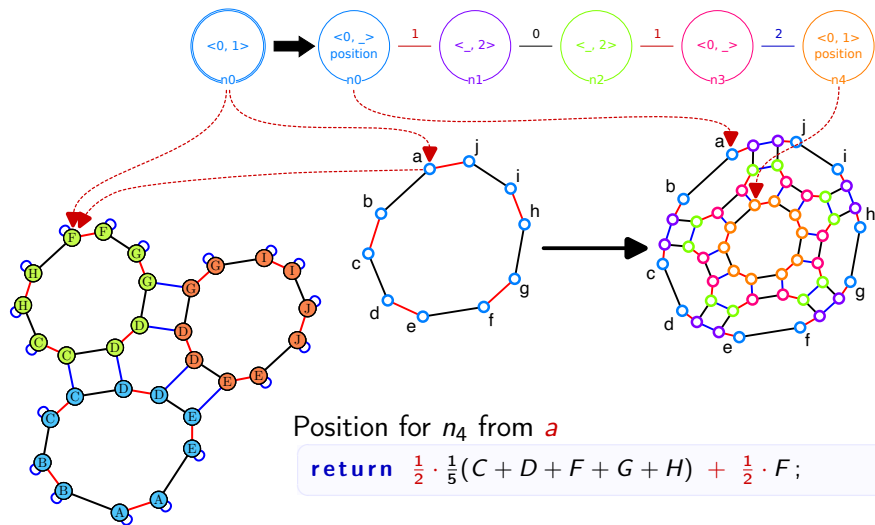
Extension to Rule Schemes



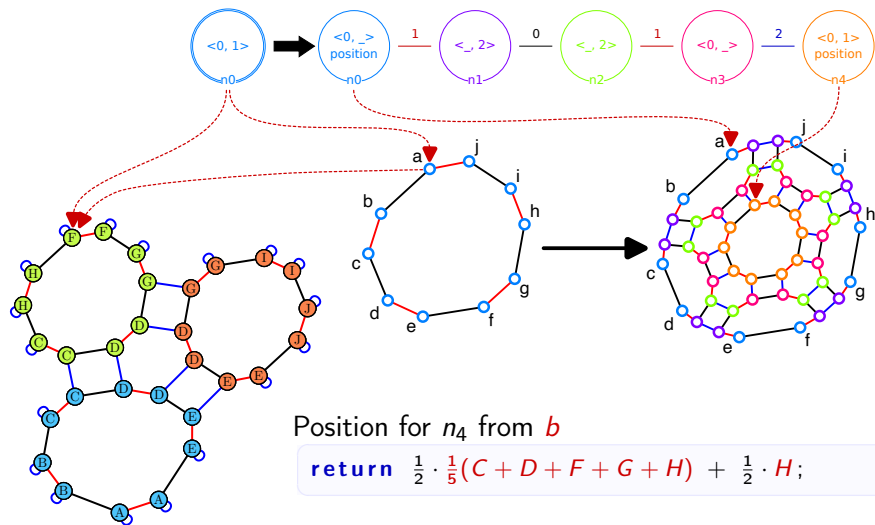
Extension to Rule Schemes



Extension to Rule Schemes

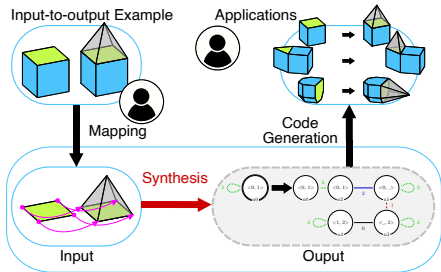


Extension to Rule Schemes

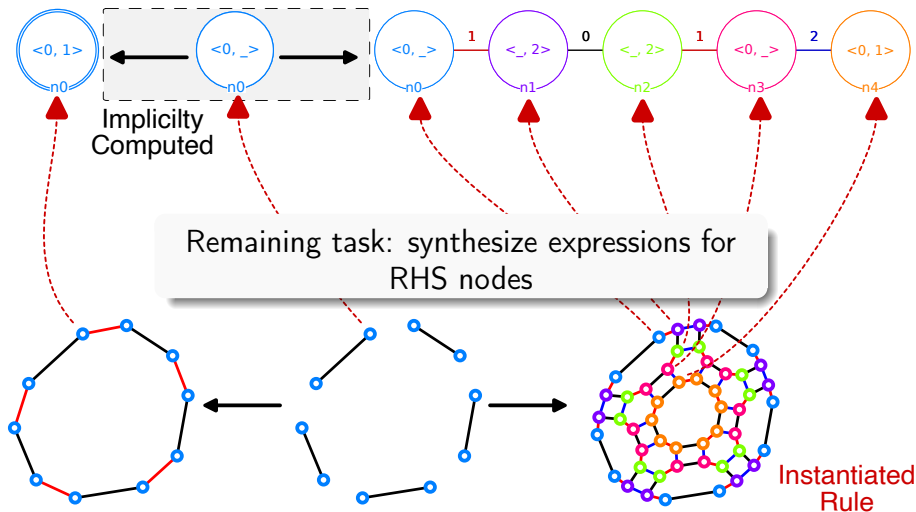


Synthesizing Geometric Operations

► How to retrieve the missing embedding expressions?

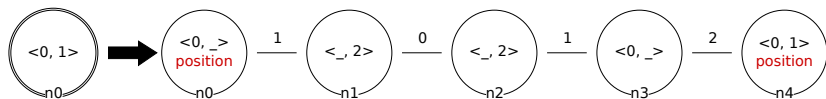


Topological Folding Algorithm¹



¹Pascual et al. 2022a.

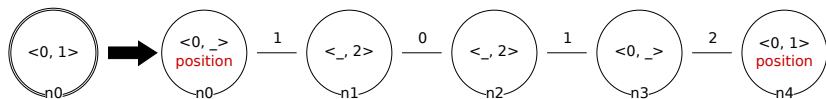
Synthesis as a Syntax-Guided Problem¹



- Where do candidate expressions come from?
- How do we pick the right one?

¹Alur et al. 2013.

Synthesis as a Syntax-Guided Problem¹



- Where do candidate expressions come from?
- How do we pick the right one?

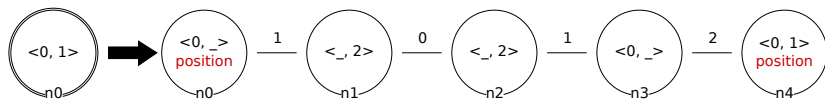
Given

- a function f , specified by a formula φ
- a language L of admissible expressions

Find an expression $e \in L$ such that $\varphi[f/e]$ is valid

¹Alur et al. 2013.

Synthesis as a Syntax-Guided Problem¹



- Where do candidate expressions come from?
- How do we pick the right one?

Given

- a function f , specified by a formula φ
- a language L of admissible expressions

Find an expression $e \in L$ such that $\varphi[f/e]$ is valid

What are L and φ ?

¹Alur et al. 2013.

Overview

Motivation: generalize independently of the input's topology

¹Gulwani et al. 2012.

²Jha et al. 2010.

³Solar-Lezama 2013.

Overview

Motivation: generalize independently of the input's topology

Program synthesis answers

- Programming-by-example¹: φ derived from input-to-output examples
- Components-based synthesis²: orbit-dependent built-in functions
- Sketch-based synthesis³: filling holes in a program

¹Gulwani et al. 2012.

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Overview

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Program synthesis answers

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Some domain-specific insights

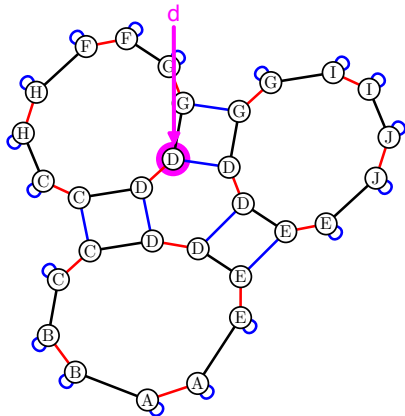
- (Generalized) barycentric coordinates can compute new positions
- Linear arithmetic is a good starting point

¹Gulwani et al. 2012.

²Jha et al. 2010.

³Solar-Lezama 2013.

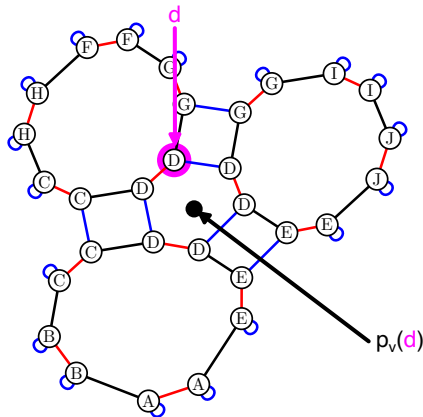
Components: Points of Interest



Components: Points of Interest

with

• p_v : vertex

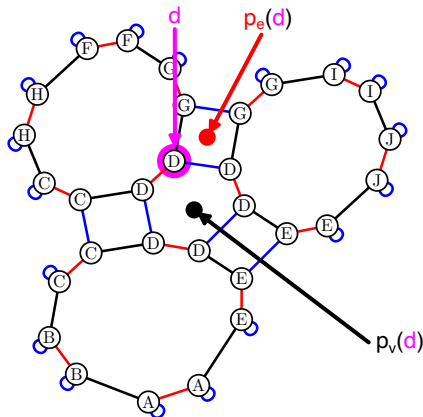


$$p_v = \text{middle}(\text{position}_{\langle \rangle}(d))$$

Components: Points of Interest

with

- p_v : vertex
- p_e : edge midpoint

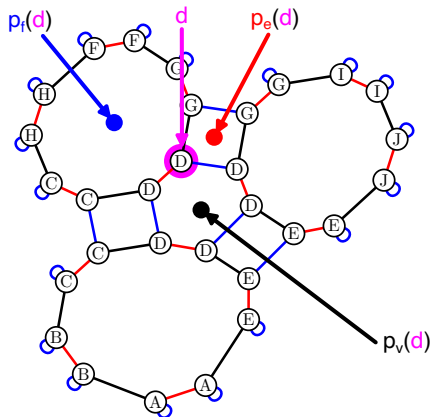


$$p_e = \text{middle}(\text{position}_{\langle 0 \rangle}(d))$$

Components: Points of Interest

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter

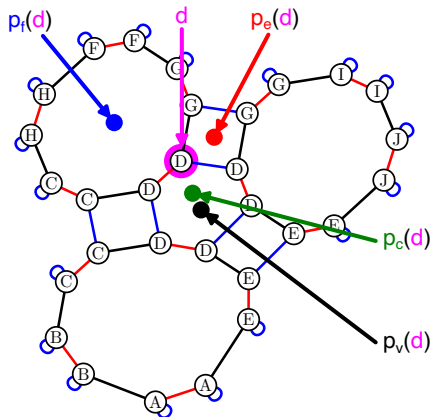


$$p_f = \text{middle}(\text{position}_{\langle 0,1 \rangle}(d))$$

Components: Points of Interest

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter
- p_s : surface barycenter



$$p_s = \text{middle}(\text{position}_{\langle 0,1,2 \rangle}(d))$$

Sketching Geometric Expressions

We fix a generic skeleton

$$\text{position}(n_R) = t + \sum w_{\text{poi}_{\langle o \rangle}(n_L)} \cdot \text{poi}_{\langle o \rangle}(n_L)$$

- $\text{poi}_{\langle o \rangle}(n_L)$: retrieved from LHS nodes
- w : weights to synthesize
- t : translation to synthesize

Sketching Geometric Expressions

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- $\text{poi}_{\langle o \rangle}(n_L)$: retrieved from LHS nodes
- w : weights to synthesize
- t : translation to synthesize

L is the set of affine expressions over the points of interest

Sketch Skeleton (?? indicates a value to be synthesized)

```
// translation
Point3 res = new Point3( ?? , ?? , ?? );

// per #node# in <1,2>-orbit of the left hand side
Point3 pV_#node# = Point3::middle(<1,2>_position( #node# ));
pV_#node#.scaleVect( ?? );
res.addVect(pV_#node#);

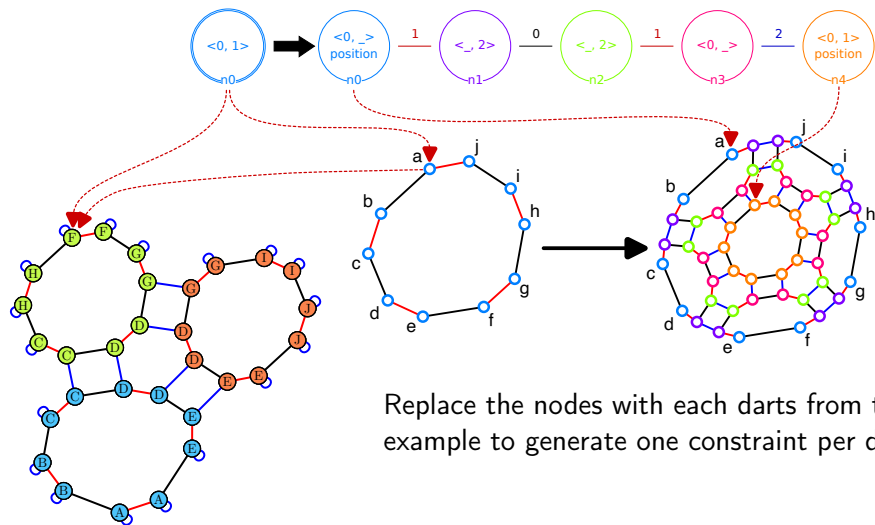
// per #node# in <0,2>-orbit of the left hand side
Point3 pE_#node# = Point3::middle(<0,2>_position( #node# ));
pE_#node#.scaleVect( ?? );
res.addVect(pE_#node#);

// per #node# in <0,1>-orbit of the left hand side
Point3 pF_#node# = Point3::middle(<0,1>_position( #node# ));
pF_#node#.scaleVect( ?? );
res.addVect(pF_#node#);

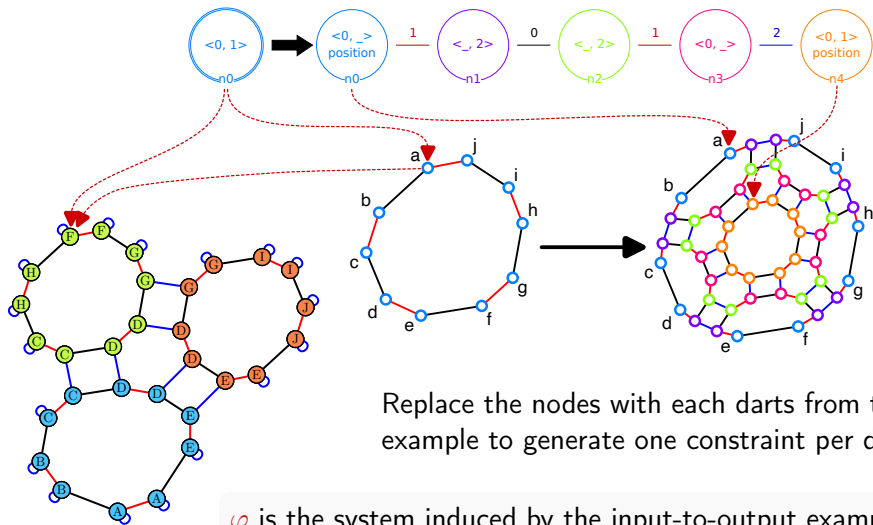
// per #node# in <0,1,2>-orbit of the left hand side
Point3 pC_#node# = Point3::middle(<0,1,2>_position( #node# ));
pC_#node#.scaleVect( ?? );
res.addVect(pC_#node#);

return res;
```

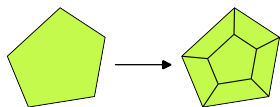
Building the Logical Specification



Building the Logical Specification



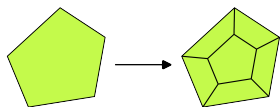
Solving the Flat Extrusion



Symbolic equation

$$\text{position}(n4) = t + w_v \cdot p_v(n0) + w_e \cdot p_e(n0) + w_f \cdot p_f(n0) + w_s \cdot p_s(n0).$$

Solving the Flat Extrusion



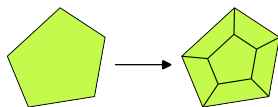
Symbolic equation

$$\text{position}(n4) = t + w_v \cdot p_v(n0) + w_e \cdot p_e(n0) + w_f \cdot p_f(n0) + w_s \cdot p_s(n0).$$

Generated system

$$\begin{cases} 0.449 = tx + w_v \cdot 0.025 + w_e \cdot 0.261 + w_f \cdot 0.874 + w_s \cdot 0.874 \\ 0.935 = ty + w_v \cdot 0.927 + w_e \cdot 0.567 + w_f \cdot 0.943 + w_s \cdot 0.943 \\ 0.685 = tx + w_v \cdot 0.496 + w_e \cdot 0.261 + w_f \cdot 0.874 + w_s \cdot 0.874 \\ 0.575 = ty + w_v \cdot 0.208 + w_e \cdot 0.567 + w_f \cdot 0.943 + w_s \cdot 0.943 \\ \vdots \qquad \qquad \qquad \vdots \end{cases}$$

Solving the Flat Extrusion



Symbolic equation

$$\text{position}(n4) = t + w_v \cdot p_v(n0) + w_e \cdot p_e(n0) + w_f \cdot p_f(n0) + w_s \cdot p_s(n0).$$

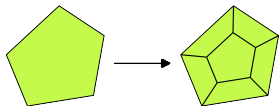
Generated system

$$\begin{cases} 0.449 = tx + w_v \cdot 0.025 + w_e \cdot 0.261 + w_f \cdot 0.874 + w_s \cdot 0.874 \\ 0.935 = ty + w_v \cdot 0.927 + w_e \cdot 0.567 + w_f \cdot 0.943 + w_s \cdot 0.943 \\ 0.685 = tx + w_v \cdot 0.496 + w_e \cdot 0.261 + w_f \cdot 0.874 + w_s \cdot 0.874 \\ 0.575 = ty + w_v \cdot 0.208 + w_e \cdot 0.567 + w_f \cdot 0.943 + w_s \cdot 0.943 \\ \vdots \\ \vdots \end{cases}$$

Delegated to a solver (Z3 with QF FP or Or-Tools with GLOP)

- $w_v = 0.5$
- $w_e = 0.0$
- $w_f = 0.5$
- $w_s = 0.0$
- $t = (0.0, 0.0)$

Synthesized Expression (Flat Extrusion)



```
// no translation
Point3 res = new Point3(0.0, 0.0, 0.0);

// vertex
Point3 p0 = Point3::middle(<1,2>_position(n0));
p0.scaleVect(0.5);
res.addVect(p0);

// face
Point3 p2 = Point3::middle(<0,1>_position(n0));
p2.scaleVect(0.5);
res.addVect(p2);

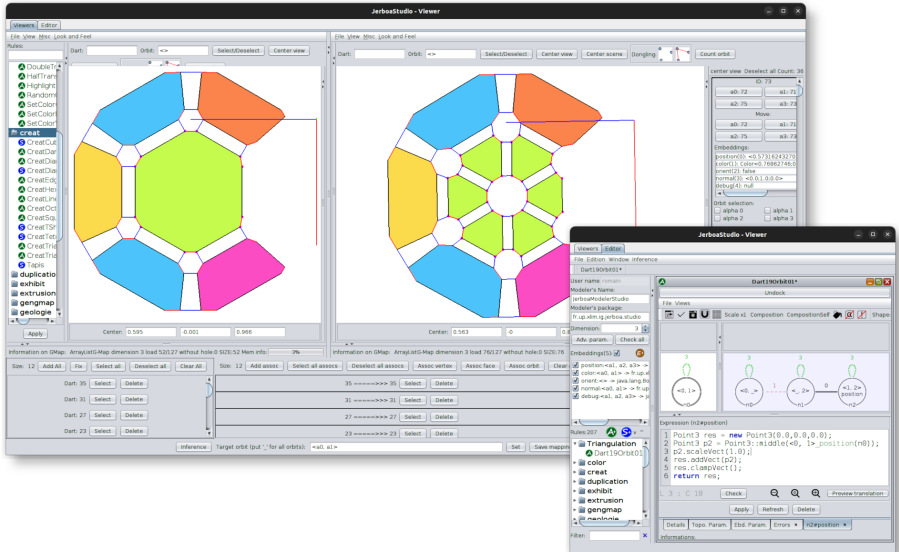
return res;
```

JerboaStudio

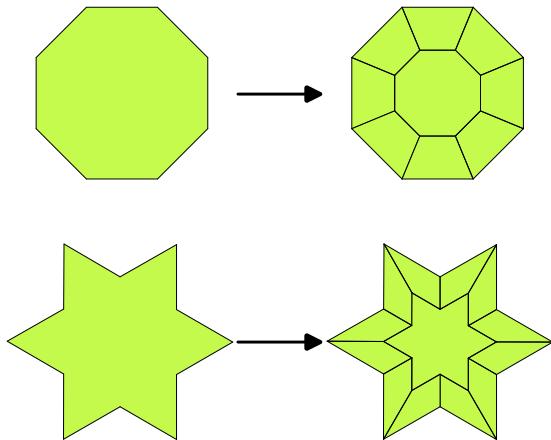
- Implementation and Evaluation



JerboaStudio (<https://gitlab.com/jerboateam/jerboa-studio>)

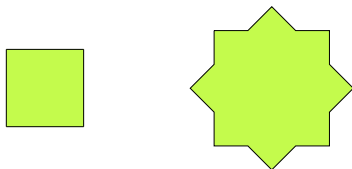


Applying the Synthesized Operation

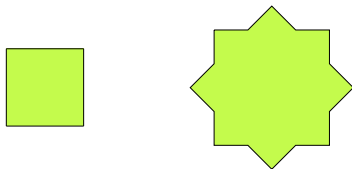


Limits

Von Koch's snowflake generated by L-systems

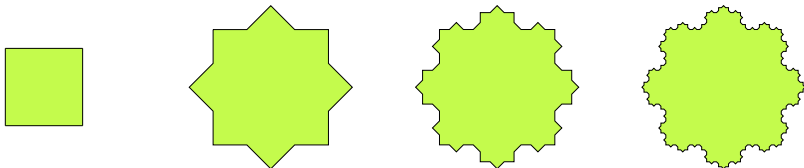


Synthesized

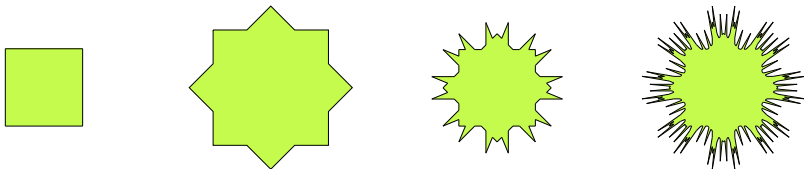


Limits

Von Koch's snowflake generated by L-systems



Synthesized



Program Synthesis for Geometric Modeling

L

φ

Program Synthesis for Geometric Modeling

L

φ

Rule level

Rule scheme

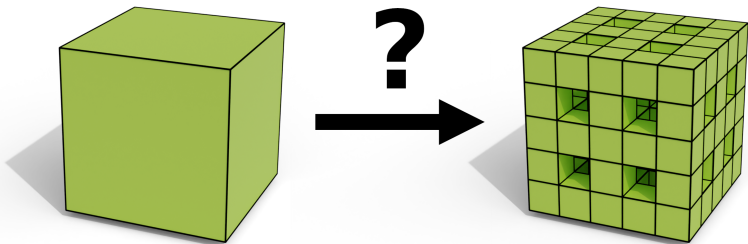
Instantiated rule

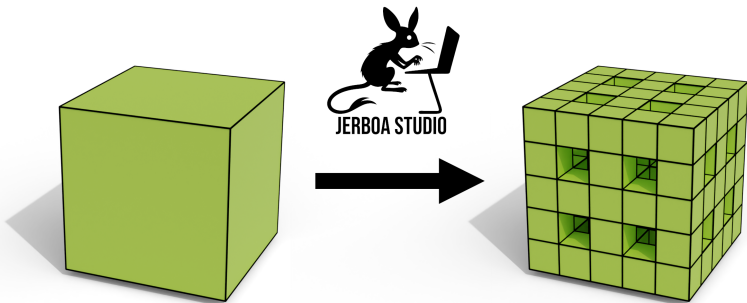
Program Synthesis for Geometric Modeling

	L	φ
Rule level	Rule scheme	Instantiated rule
Corresponds to	Affine combinations of points of interest	Concrete system derived from the example

Program Synthesis for Geometric Modeling

	L	φ
Rule level	Rule scheme	Instantiated rule
Corresponds to	Affine combinations of points of interest	Concrete system derived from the example
Extend with	• other points of interest • other computations	• multi-examples • counter-examples

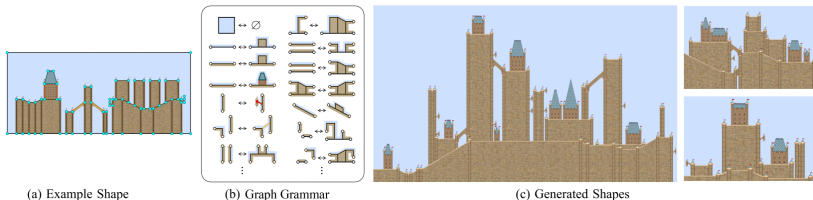




Synthesis in Geometric Modeling

1. Object generation:

- Inverse procedural modeling: retrieving parameters.¹
- L-systems: retrieving formal rules.²
- Constructive solid geometry: retrieving sequences of operations.³
- Polyhedral decomposition: retrieving a graph grammar. Illustration from (Merrell 2023).



¹Wu et al. 2014; Emilien et al. 2015.

²Santos et al. 2009; Št'ava et al. 2010; Guo et al. 2020.

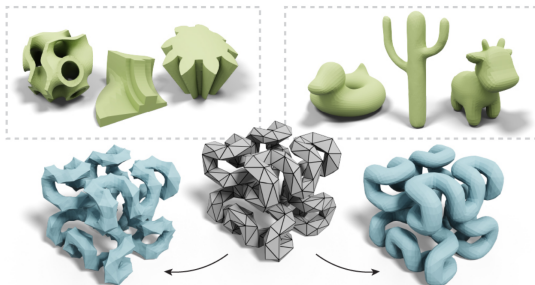
³Sharma et al. 2018; Kania et al. 2020; Xu et al. 2021.

Synthesis in Geometric Modeling

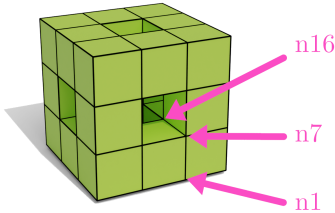
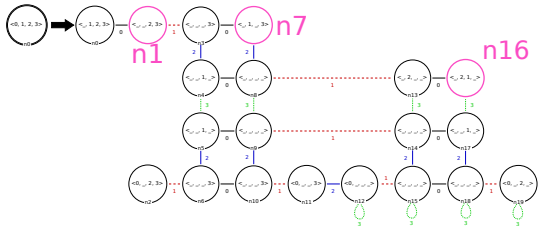
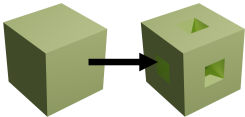
1. Object generation

2. Pure geometry:

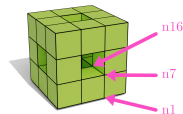
- Retrieve non-linear weights of a Loop-based subdivision scheme for mesh refinement. Illustration from (Liu et al. 2020).



Menger Sponge



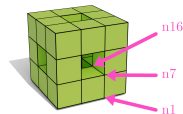
Menger Sponge



Node $n7$

```
Point3 res = new Point3(0.0,0.0,0.0);  
Point3 p0 = Point3::middle(<>_position(n0));  
p0.scale(0.33333333134651184);  
res.addVect(p0);  
Point3 p2 = Point3::middle(<0,1>_position(n0));  
p2.scale(0.66666666865348816);  
res.addVect(p2);  
return res;
```

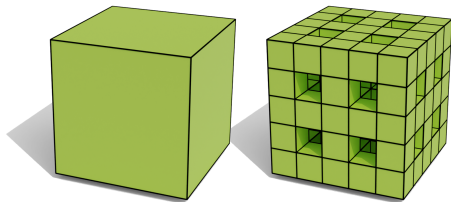
Menger Sponge



Node *n16*

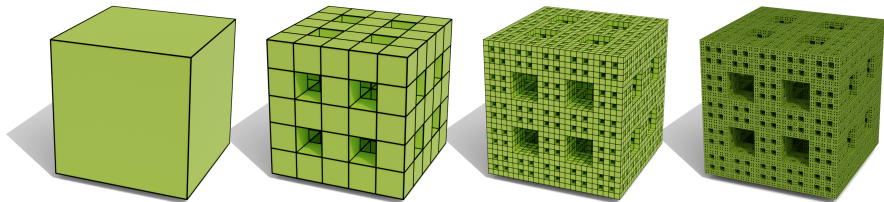
```
Point3 res = new Point3(0.0,0.0,0.0);  
Point3 p0 = Point3::middle(<>_position(n0));  
p0.scale(0.33333333134651184);  
res.addVect(p0);  
Point3 p3 = Point3::middle(<0,1,2>_position(n0));  
p3.scale(0.6666666865348816);  
res.addVect(p3);  
return res;
```

$(2, 2, 2)$ -Menger Polycube¹



¹Richaume et al. 2019.

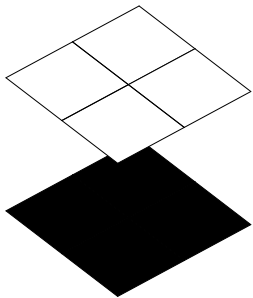
$(2, 2, 2)$ -Menger Polycube¹



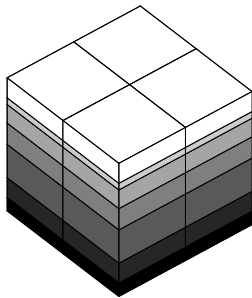
¹Richaume et al. 2019.

Geology inspired

Before



After



Positions and colors

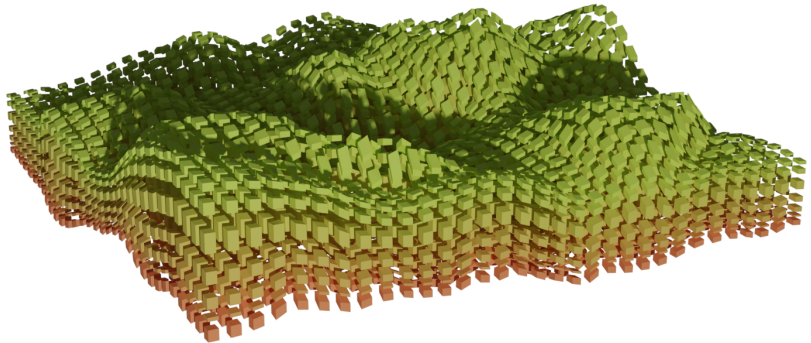
Geology inspired

Before



Geology inspired

After



Evaluation: Benchmark on Subdivision Schemes

Same input (cube) ensures each sketch has **8 unknowns, 48 eqs.**

Java 11, OR-Tools 9.6, Intel Ultra 7 @ 4.8GHz, 32GB RAM.

Operation	# Expr	# Synth (%)	# Correct (%)	SolT (ms)	SynT (ms)
Surface Subdivisions					
Catmull-Clark	3	3 (100%)	1 (33%)	1.2	10
Doo-Sabin	1	1 (100%)	0 (0%)	0.4	11
Powell-Sabin	2	2 (100%)	2 (100%)	0.9	11
Blender Subdivide	2	2 (100%)	2 (100%)	0.9	9
Sierpinski Carpet	2	2 (100%)	2 (100%)	1.0	13
$\sqrt{3}$	2	2 (100%)	1 (50%)	0.9	11
Volume Subdivisions					
Menger	3	3 (100%)	3 (100%)	1.4	26
(2,2,2)-Menger	9	9 (100%)	9 (100%)	2.9	64
Surface to Volume					
Mesh to Tet	1	1 (100%)	1 (100%)	0.5	12
Mesh to Hex	3	3 (100%)	3 (100%)	1.2	16

24/28 expressions synthesized (failures outside of search space)

Full pipeline: 0.5–2 s

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