Karlsruhe Institute of Technology

Formal Foundations of Consistency in Model-Driven Development

KeY Symposium 2024

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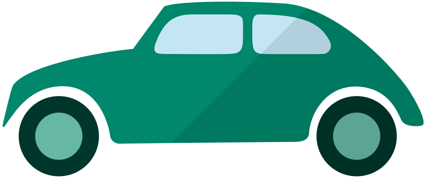
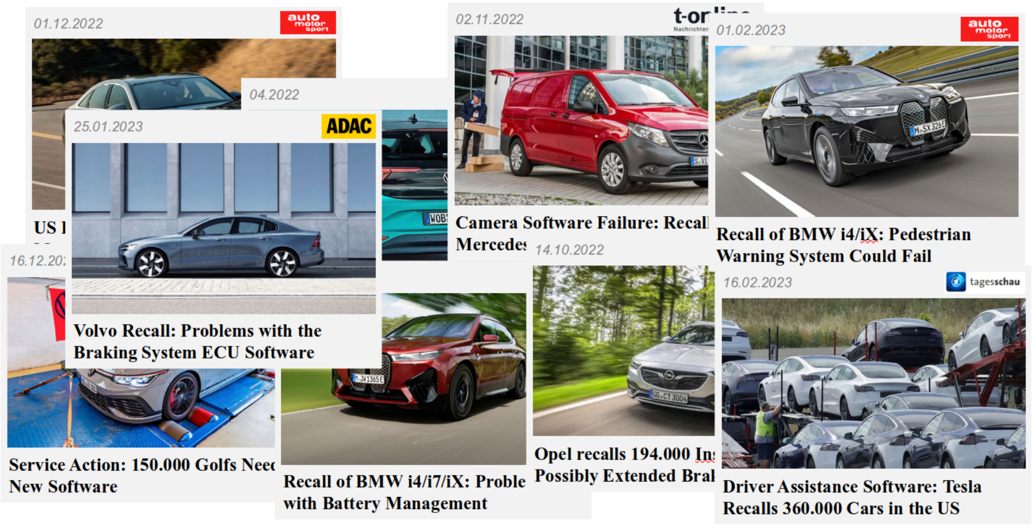
Convide[1]

Consistency in the **Vi**ew-Based **De**velopment of Cyber-Physical Systems

- Software engineering
- Mechanical engineering
- Electrical engineering
- Formal methods



[1] Sonderforschungsbereiche (SFB) financed by the Deutsche Forschungsgemeinschaft (DFG)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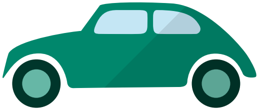
tagesschau
Driver Assistance Software: Tesla Recalls 360.000 Cars in the US


HAVE YOU TRIED
TURNING IT OFF AND ON AGAIN?
quickmeme.com

The Virtual Single Underlying Model methodology (V-SUM)



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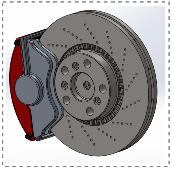
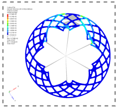
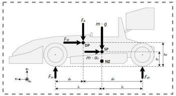


3D Brake Assembly View

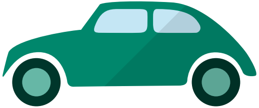


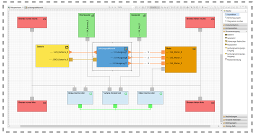
Tribological Simulation View

X-in-the-Loop Test Deployment View

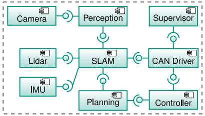


Simplified Vehicle Model View





E/E Topology View (PREEvision)



Autonomous Driving Components View

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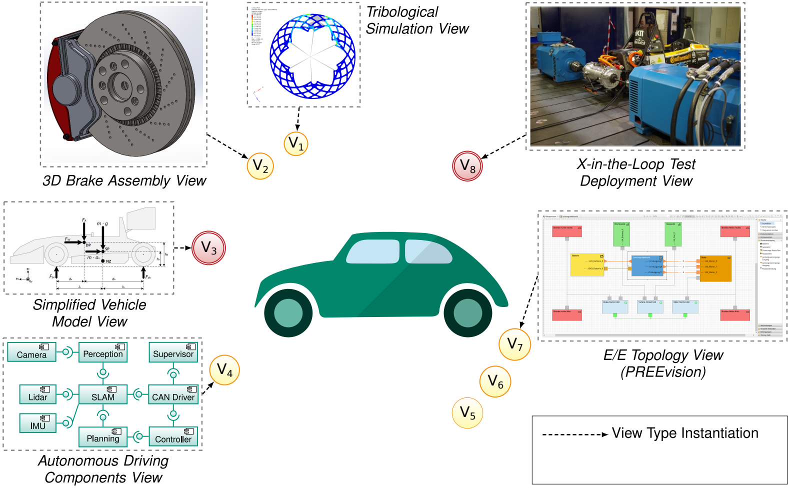

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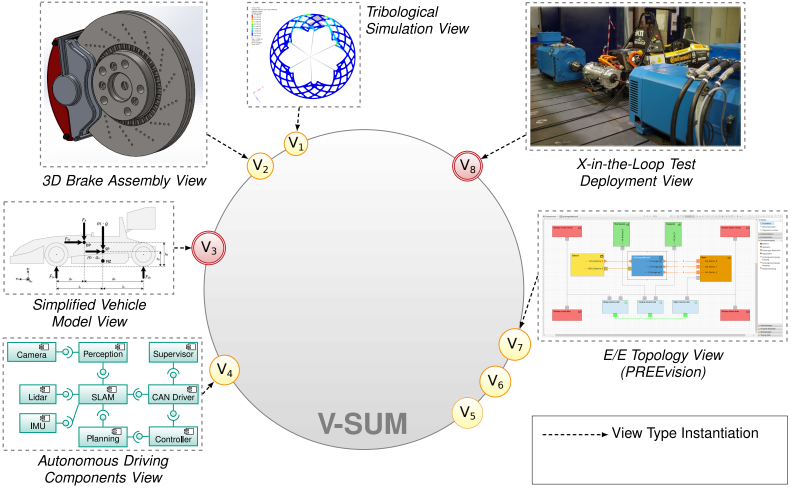

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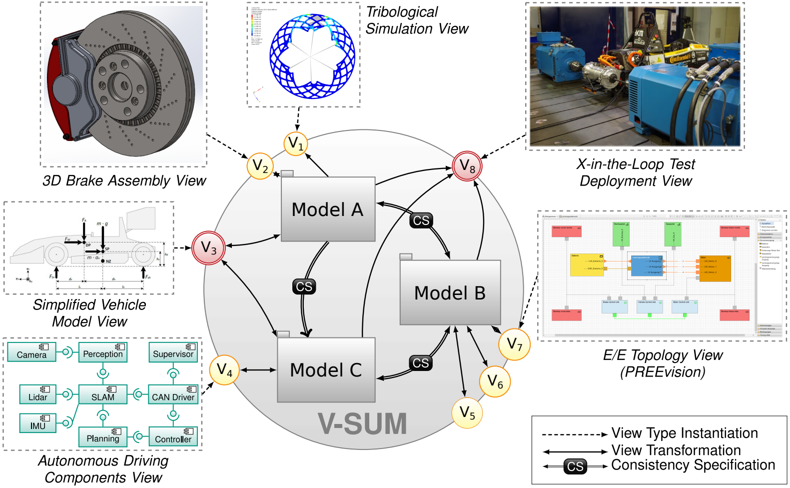

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An abstract representation of an original entity (for a given purpose)

- Anything you may write with UML
- An automaton
- A Petri net
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The set of syntactically admissible models is described by a **meta-model**, e.g., a formal grammar

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We abstract **consistency** as a relation between models

Formalizing the V-SUM approach

- A set-theoretic approach to V-SUM consistency

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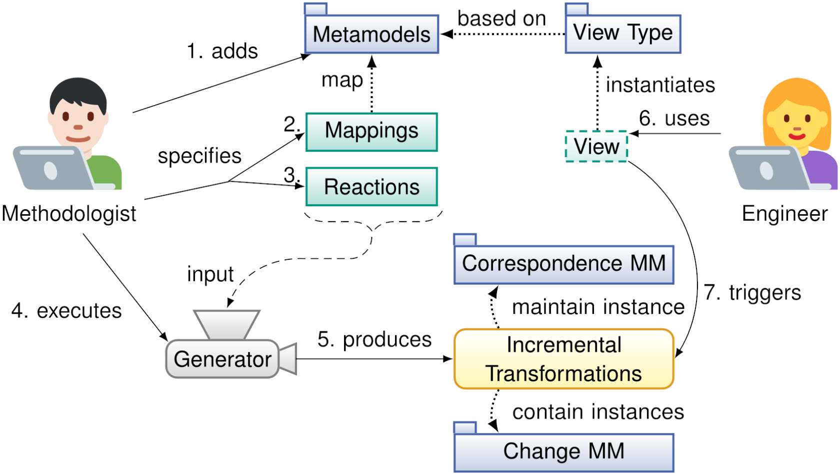
- A V-SUM model m is **consistent** wrt. CR if $m \in CR$, written $CR(m)$

How is consistency specified?

- The Vitruvius approach

Consistency Preservation with Vitruvius



Consistency from semantics

- Semantical V-SUM

An abstract notion of semantics

Examples of semantics

- **Satisfying structures** in Tarskian approach to logic,
- **Denotational** or **operational semantics** of programming languages
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It is purpose-dependent, but the choice of S does not matter

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Assume each meta-models M_i is equipped with an abstract semantics $[\![\cdot]\!]_i : M_i \to S_i$,
a **semantic consistency relation** is a relation $SCR \subseteq \prod_{i \leq n} S_i$

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We obtain a consistency relation CR_{SCR} on $\prod_{i \leq n} M_i$ (and therefore a V-SUM meta-model)

Examples

for $[\![\cdot]\!]_i$ *and* S_i

- the **set of satisfying structures** (Tarskian approach to logic)
- the **result of some tests** on a mechanical part
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Allows for user-defined semantics and relations

Reasoning on semantics

- A little bit of lattice theory

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The models m_1 and m_2 are equal modulo $[\![\cdot]\!]$: $m_1 \equiv m_2 \iff [\![m_1]\!] = [\![m_2]\!]$

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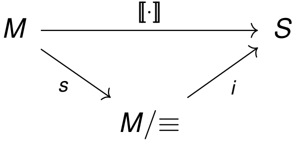

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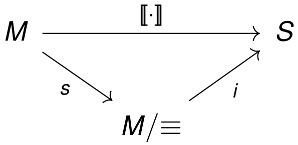

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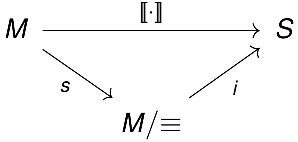

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Reduces the study to the quotient set M/R for the equivalence relations $R \subseteq M \times M$

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> The choice of representatives (or names) is irrelevant to compare the amount of information kept by the abstract semantics

The lattice of semantics

Theorem (Crawley and Dilworth 1973, Chap. 12 or Grätzer 2003, Sect. IV.4)

The set of all equivalence relations on a set form a complete lattice called the **equivalence lattice** with set-inclusion as order

- Meet (infimum): $\bigwedge R = \bigcap R$
- Join (supremum): $\bigvee R = (\bigcup R)^*$

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The isomorphism transfers the lattice structure from the equivalence relations to the abstract semantics, reserving the order:

$$M/R_1 \sqsubseteq M/R_2 \iff R_2 \subseteq R_1$$

We write $\mathcal{L}_{\mathrm{sem}}^{M}$ for the lattice of semantics on M

Intuitions

Given two semantics $[\![\cdot]\!]_1$ and $[\![\cdot]\!]_2$, $[\![\cdot]\!]_1 \sqsubseteq [\![\cdot]\!]_2$ if and only if $[\![\cdot]\!]_2$ allows distinguishing between the same model as $[\![\cdot]\!]_1$ and possibly more

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The **bottom element** $[\![\cdot]\!]_\perp : M \to M/M^2 \simeq \{\star\}$ in the lattice of semantics corresponds to the trivial relation M^2 that relates any two elements

- All models have the same semantics $[\![m]\!]_\perp = \star$

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The **bottom element** $[\![\cdot]\!]_\bot : M \to M/M^2 \simeq \{\star\}$ in the lattice of semantics corresponds to the trivial relation M^2 that relates any two elements

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The **top element** $[\![\cdot]\!]_\top : M \to M/\mathrm{id}_M \simeq M$ corresponds to the identity relation that relates every element only to itself

- Every model $m \in M$ is its own semantic value $[\![m]\!]_\top = m$

Compatible semantics

A family of abstract semantics $(\llbracket\cdot\rrbracket_i : M_i \to S_i)_{i \leq n}$ is **compatible** with CR if and only if there is a semantic consistency relation $SCR \subseteq \prod_{i \leq n} S_i$ st.

$$CR = CR_{SCR}$$

Compatible semantics encode enough information to determine if models are consistent

Natural semantics

We consider the relation $\sim_i$ st. models are related if and only if the sets of tuples that extend them to consistent V-SUM models are the same:

$$m_a \sim_i m_b \iff CR^{\nabla i}(m_a) = CR^{\nabla i}(m_b)$$

with

$$CR^{\nabla i}(\nu) = \left\{ (m_1, \ldots, m_{i-1}, m_{i+1}, \ldots m_n) \in \prod_{j \neq i} M_j \mid CR(m_1, \ldots, m_{i-1}, \nu, m_{i+1}, \ldots m_n) \right\}$$

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Natural semantics contains just the information needed to compute *CR*

Results

Proposition 1

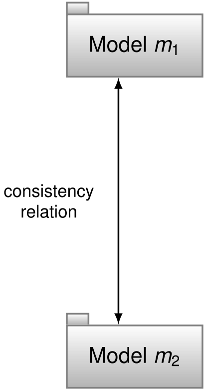
The natural semantics are compatible with *CR*

Proposition 2

Semantics compatible with *CR* form complete lattices with the natural semantics as bottom elements

Proof: By considering $SCR^{\mathrm{nat}} = \{(\llbracket m_1 \rrbracket_1^{\mathrm{nat}}, \ldots, \llbracket m_n \rrbracket_n^{\mathrm{nat}}) \mid CR(m_1, \ldots, m_n)\}$ and the quotient sublattice (see Crawley and Dilworth 1973, Chap. 2)

Conclusion



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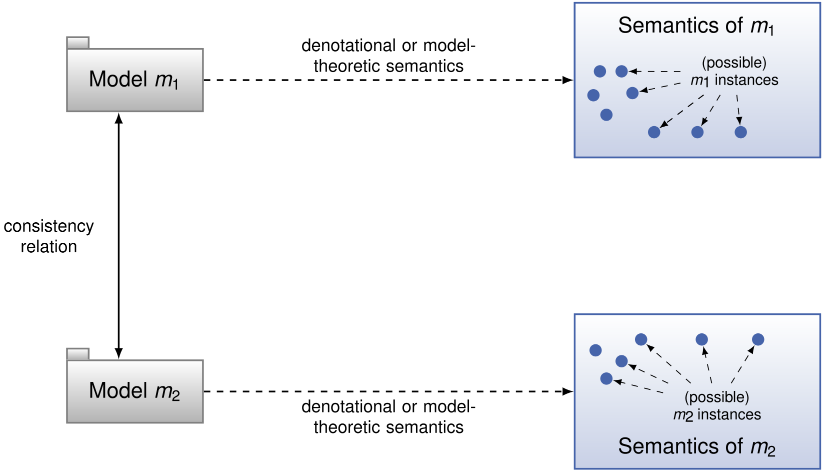

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denotational or model-theoretic semantics
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(possible) m_1 instances
consistency relation
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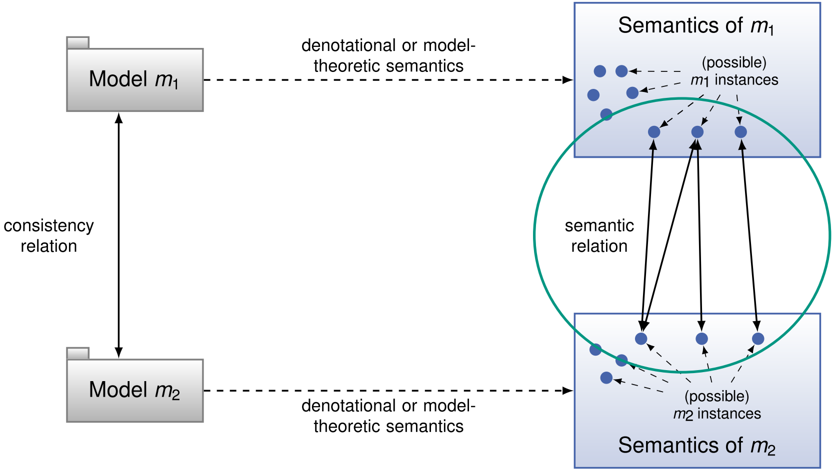
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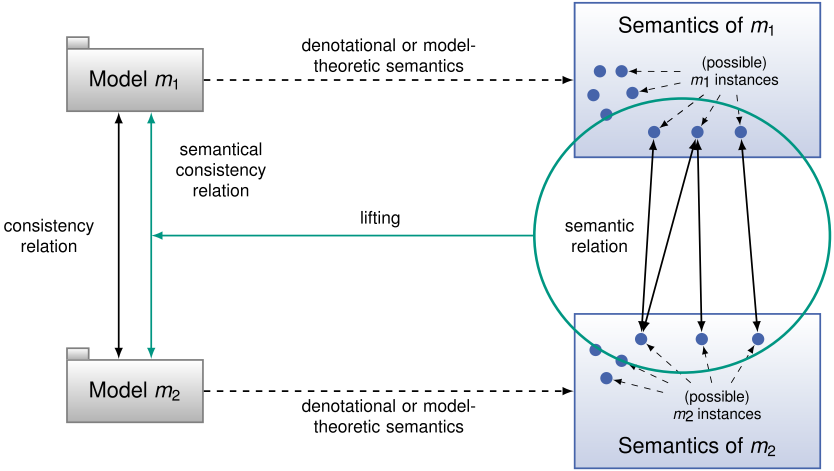

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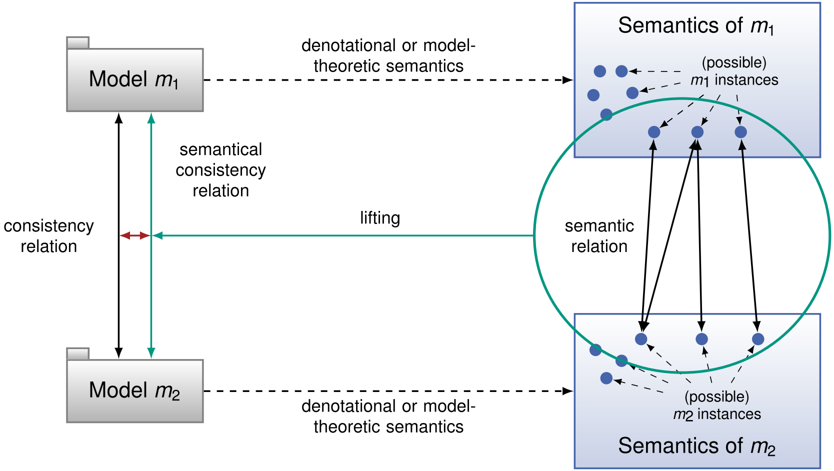

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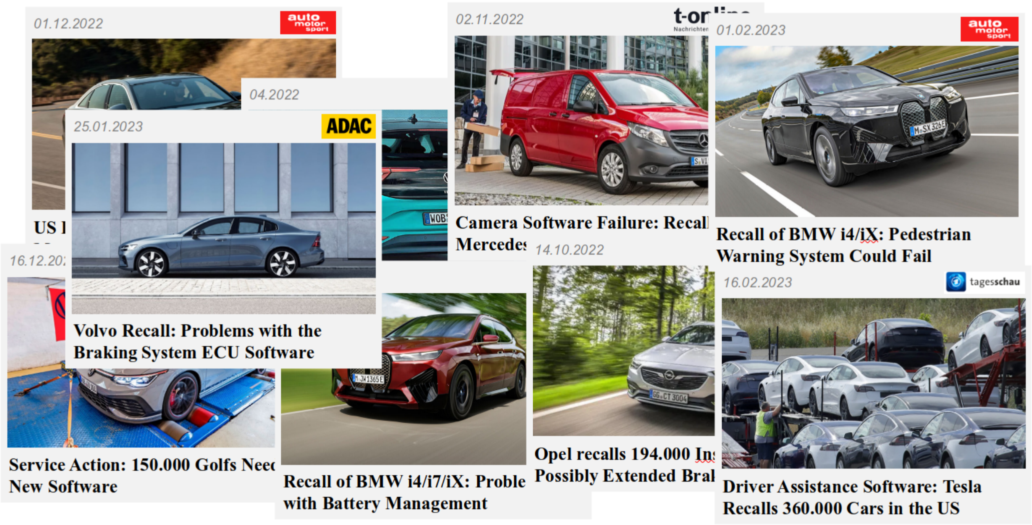

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Service Action: 150.000 Golfs Need New Software

Recall of BMW i4/i7/iX: Proble with Battery Management

02.11.2022

Camera Software Failure: Recall Mercede

14.10.2022

Opel recalls 194.000 In Possibly Extended Bral

01.02.2023

Recall of BMW i4/iX: Pedestrian Warning System Could Fail

16.02.2023

Driver Assistance Software: Tesla Recalls 360.000 Cars in the US


HAVE YOU TRIED
TURNING IT OFF AND ON AGAIN?
quickmeme.com

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