Inference of geometric modeling operations

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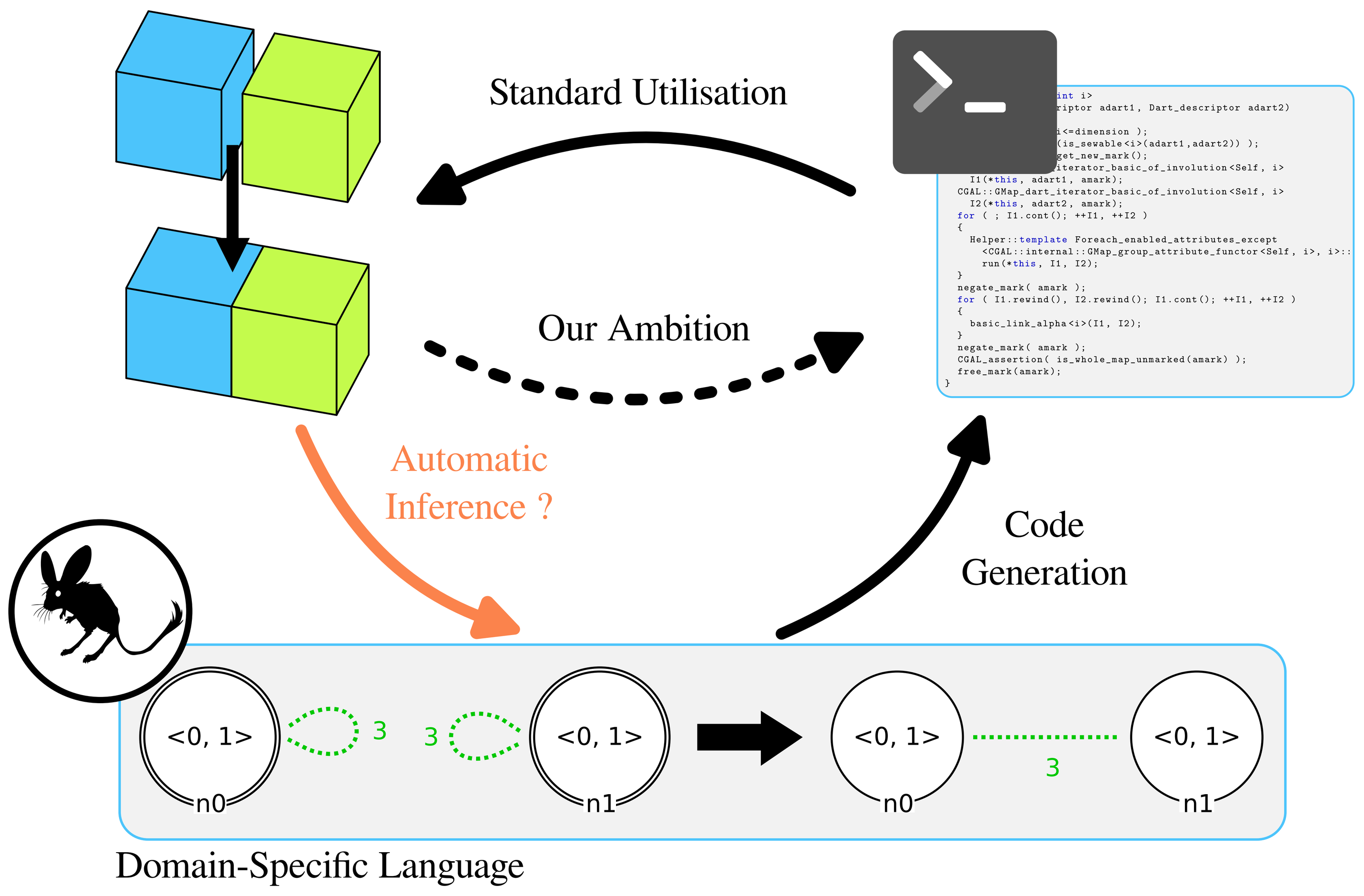
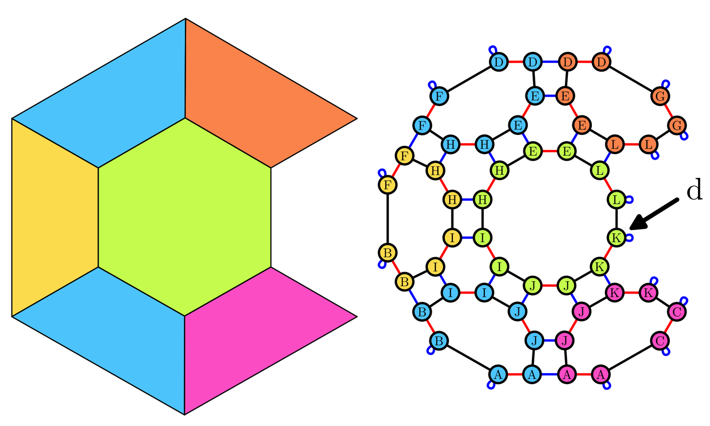
1. Problematic and motivating context

Conceiving an operation can be a difficult ask even though it may be easily described from an example. We propose a new paradigm exploiting a **domain specific language** as a guideline to retrieve an operation from an object before and after modification.



2. Object representation: embedded generalized maps

Topology-based geometric modeling uses combinatorial structures to encode the decomposition of an object's topological cells (vertices, edges, faces . . .) which is embedded in a geometric space to store additional informations.



Generalized maps (Gmaps) [2] built as graphs:

- graph: incidence relations between the cells
- node attributes: geometric embedding values

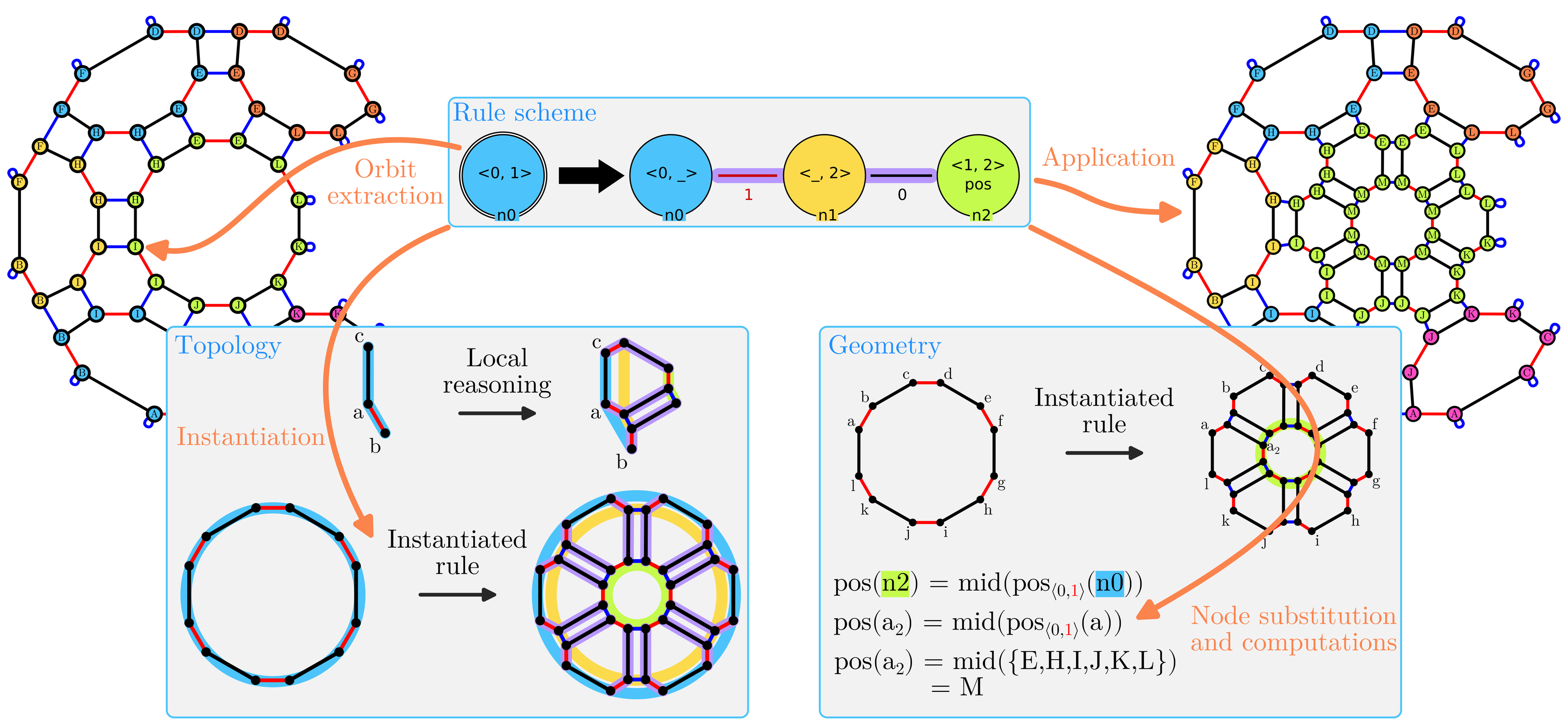
Orbits encode the object's topological cells as subgraphs:

- positions on vertices (orbits $\langle 1, 2 \rangle$).
- colors on faces (orbits $\langle 0, 1 \rangle$).

Color legend: 0, 1, 2.

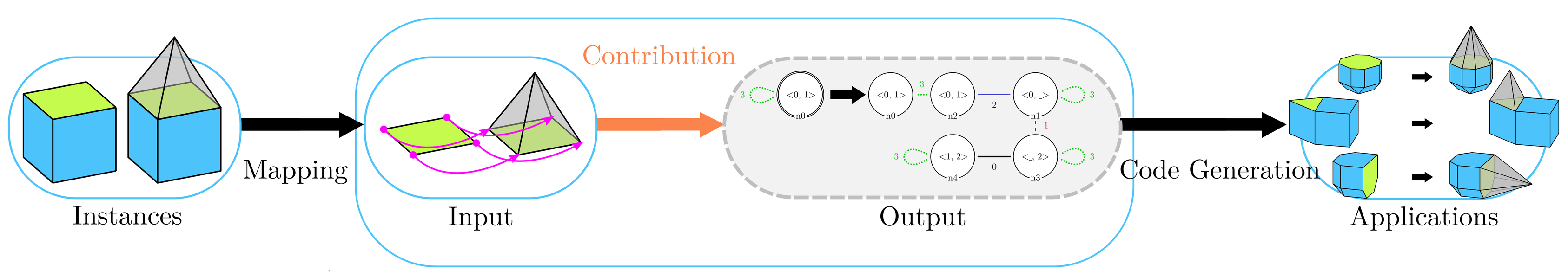
3. Modeling operations formalized as graph transformation [5, 1]

Graph transformation [3] extends rewriting to non-linear structures, allowing to formalize modifications of embedded Gmaps: a graph product extends rules to abstract over the underlying topology while algebraic data types and algebra morphisms describe geometric modifications.



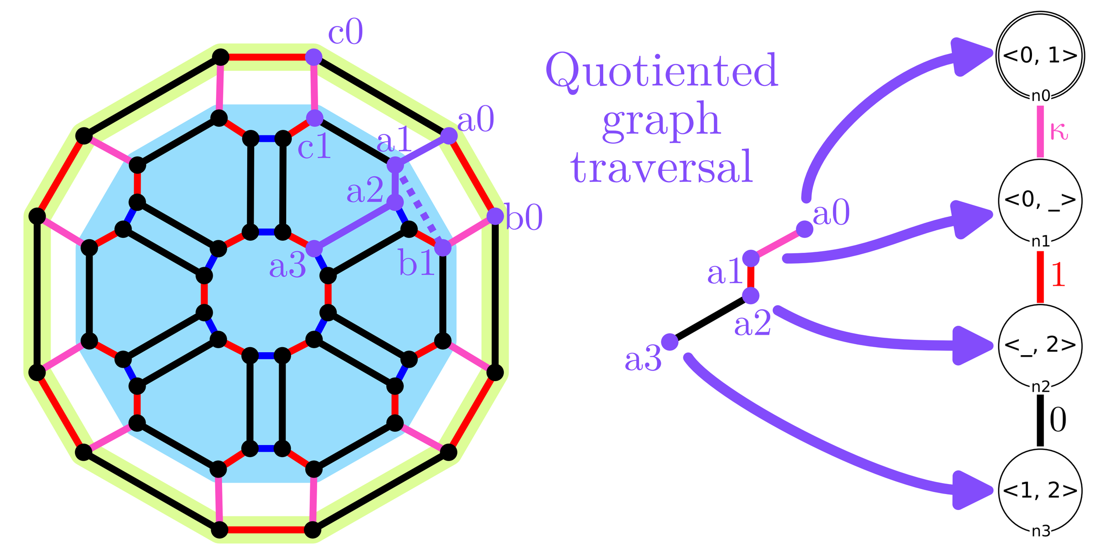
4. Method and global workflow

General idea: reverse the instantiation and application process.



5. Topological folding algorithm [4]

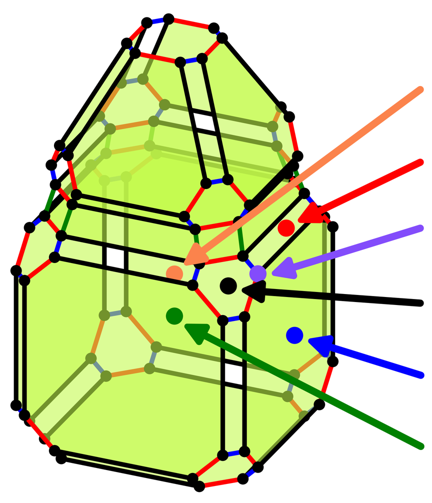
Input: two partial Gmaps with preservation relation (arcs κ), an orbit type $\langle o \rangle$ and a dart a.
Output: a graph $\mathcal{S}$ encoding the rule on $\langle o \rangle$, given that the operation is applied to a.
General idea: quotiented graph traversal algorithm iteratively folding nodes and arcs.



Color legend: 0, 1, 2, κ. Before instance in blue, after instance in green.

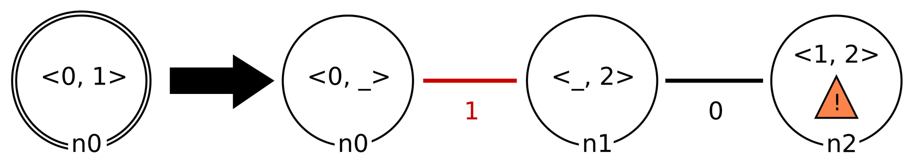
6. Retrieving geometric expressions: linear hypothesis

The output of the topological folding algorithm lacks expressions for the geometric computation.
Problem: Rules abstract topological cells meaning darts will share the same expression.
Solution: Values of interest that exploit the topology to express computations independently from the actual structure.



$p_{cc}(d) = mid(pos_{\langle 0,1,2,3 \rangle}(d))$ connected component barycenter
$p_e(d) = mid(pos_{\langle 0 \rangle}(d))$ edge midpoint
d
$p_v(d) = pos(d)$ vertex position
$p_f(d) = mid(pos_{\langle 0,1 \rangle}(d))$ face barycenter
$p_s(d) = mid(pos_{\langle 0,1,2 \rangle}(d))$ volume barycenter

Triangulation operation:



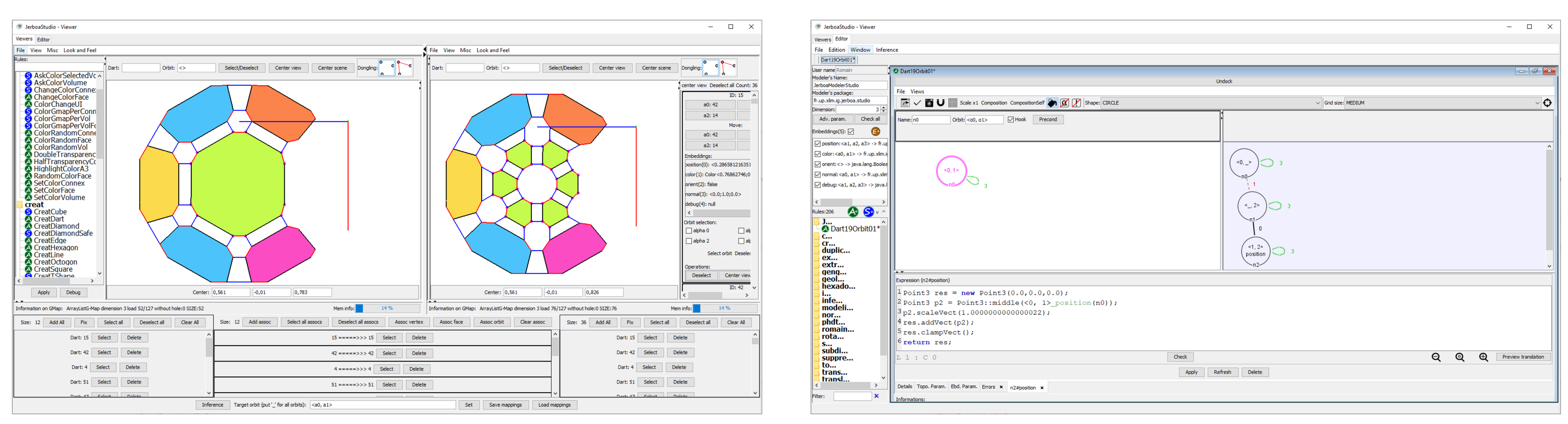
We obtain the following equation, which we solve as a constraint satisfaction problem.

$$pos(n_2) = \underbrace{w_v p_v(n_0)}_{vertex} + \underbrace{w_e p_e(n_0)}_{edge} + \underbrace{w_f p_f(n_0)}_{face} + \underbrace{w_s p_s(n_0)}_{volume} + \underbrace{w_{cc} p_{cc}(n_0)}_{cc} + t$$

Solution found: $w_f = 1.0$ with all other variables being zero.

7. JerboaStudio

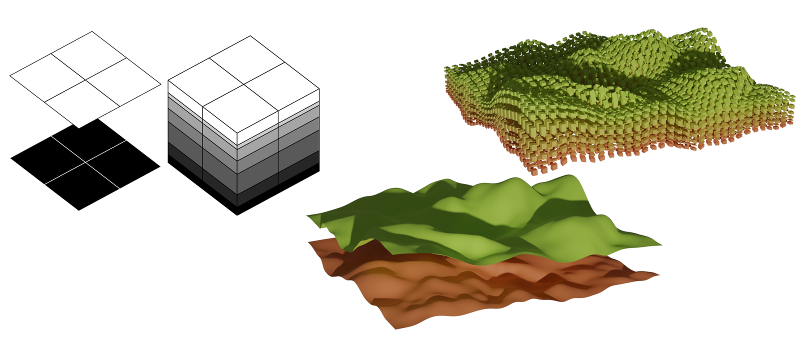
The **viewer** tab (left) contains both instances used for the inference while the **editor** tab (right) allows visualizing (and modifying) the inferred operations.



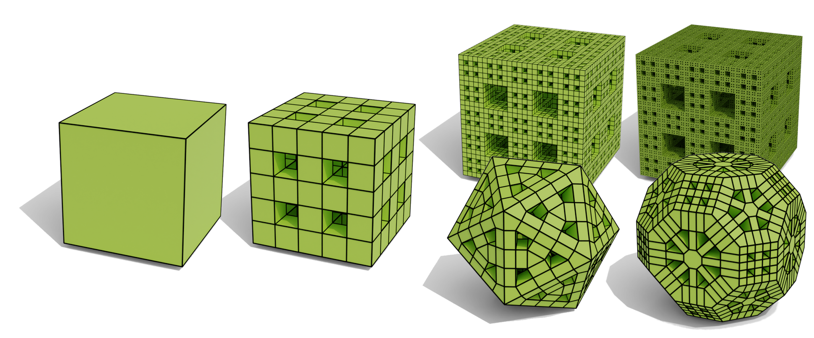
8. Results and conclusion

Main results : proof of correctness for topological folding algorithm [4], but some operations cannot be expressed directly in the language ; retrieve computations with any embedding in a vectorial space by many cases remains out of the linear hypothesis.

Application to layering:



Menger $(2, 2, 2)$ sponge [6]:



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[4] R. Pascual, H. Belhaouari, A. Arnould, and P. Le Gall. Inferring topological operations on generalized maps: Application to subdivision schemes. *GVC*, 2022.
[5] R. Pascual, P. Le Gall, A. Arnould, and H. Belhaouari. Topological consistency preservation with graph transformation schemes. *SCP*, 2022.
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GTMG : Groupe de Travail Modélisation Géométrique

La présentation de ces travaux a été primée lors des journées du GT en Mars 2022.