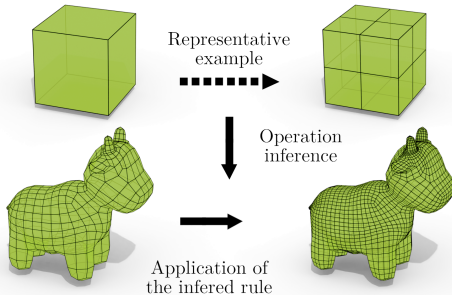


Inference of geometric modeling operations

Romain Pascual

joint work with Pascale Le Gall,
Hakim Belhaouari, and
Agnès Arnould

March 3, 2023



université
PARIS-SACLAY



Geometric modeling

► How to realize such a scene?

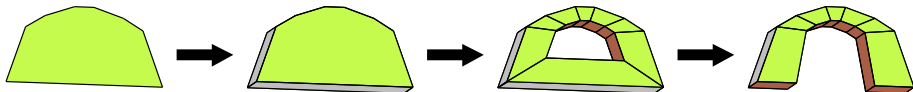


Geometric modeling

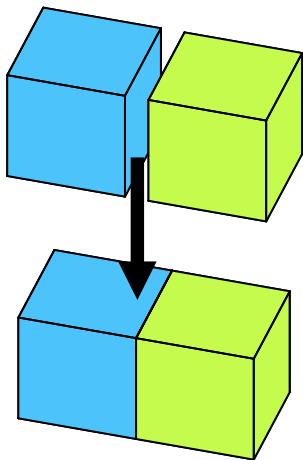
► How to realize such a scene?



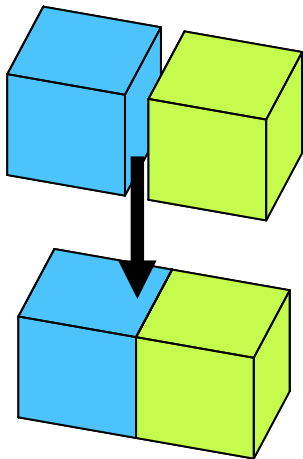
► Creating objects



Designing modeling operations



Designing modeling operations

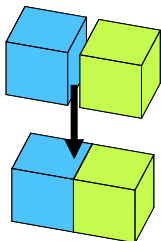


CGAL's sew operation

```
template<unsigned int i>
void sew(Dart_descriptor adart1, Dart_descriptor adart2)

{
    CGAL_assertion( i<=dimension );
    CGAL_assertion( (is_sevable<i>(adart1,adart2)) );
    size_type amark=get_new_mark();
    CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
        I1(*this, adart1, amark);
    CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
        I2(*this, adart2, amark);
    for ( ; I1.cont(); ++I1, ++I2 )
    {
        Helper::template Foreach_enabled_attributes_except
            <CGAL::internal::GMap_group_attribute_functor<Self, i>, i>::
                run(*this, I1, I2);
    }
    negate_mark( amark );
    for ( I1.rewind(), I2.rewind(); I1.cont(); ++I1, ++I2 )
    {
        basic_link_alpha<i>(I1, I2);
    }
    negate_mark( amark );
    CGAL_assertion( is_whole_map_unmarked(amarck) );
    free_mark(amarck);
}
```

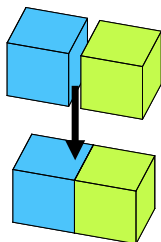
Inferring modeling operations



Standard Approach

[illegible]

Inferring modeling operations



Standard Approach



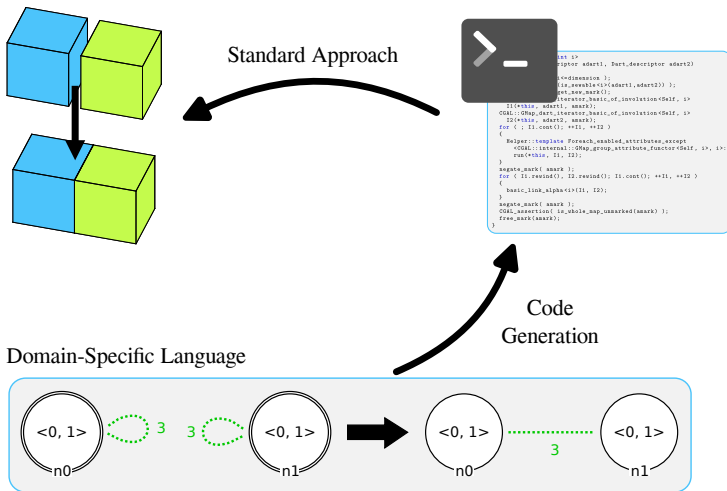
Our Ambition



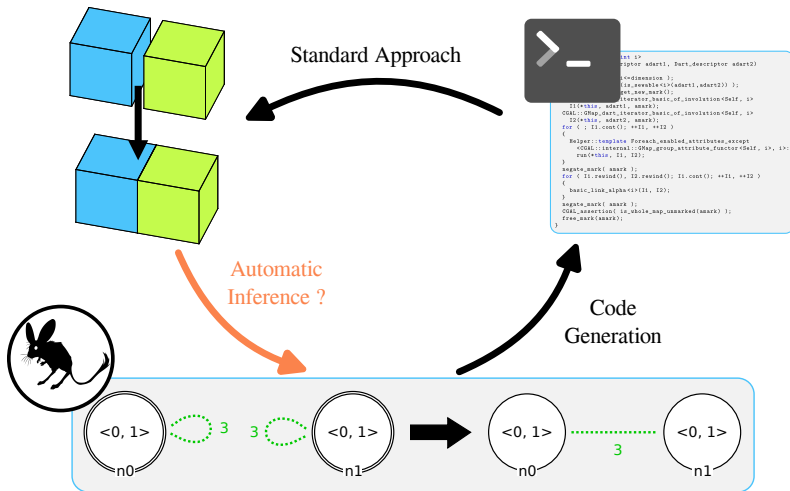
```

    int i>
    descriptor adart1, Dart_descriptor adart2)
    {
        i<<dimension;
        if (is_sevable<i>(adart1,adart2)) );
        get_new_mark();
        Iterator basic_of_involation<Self, i>
        II(*this, adart1, amark);
        CGL::OMap_dart_iterator basic_of_involation<Self, i>
        I2(*this, adart2, amark);
        for ( ; I1.cont(); ++I1, ++I2 )
        {
            Helper::template ForEach_enabled_attributes_except
            <CGL::Interval> OMap_group_attributes_function<Self, i>, i>;
            run(*this, I1, I2);
        }
        segregate_mark( amark );
        for ( I1.reset(); I1.cont(); ++I1, ++I2 )
        {
            basic_link_alpha<i>(I1, I2);
        }
        segregate_mark( amark );
        CGL::assertion( is_whole_map_unmarked(amark) );
        free_mark(amark);
    }
  
```

Inferring modeling operations



Inferring modeling operations



Strengths and weaknesses of Jerboa's DSL

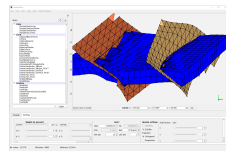
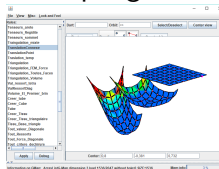
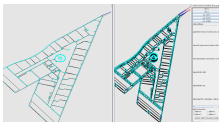
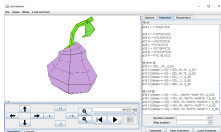
► Main characteristics:

- Dedicated to Gmaps¹
- Based on graph rewriting²



► Successful applications:

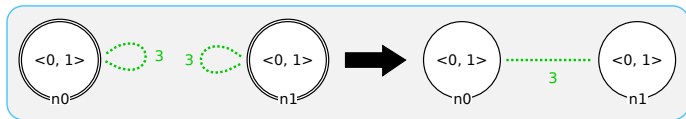
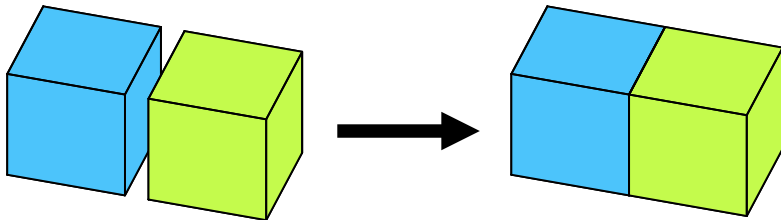
- Plant growth
- Architecture
- Spring-mass
- Geology



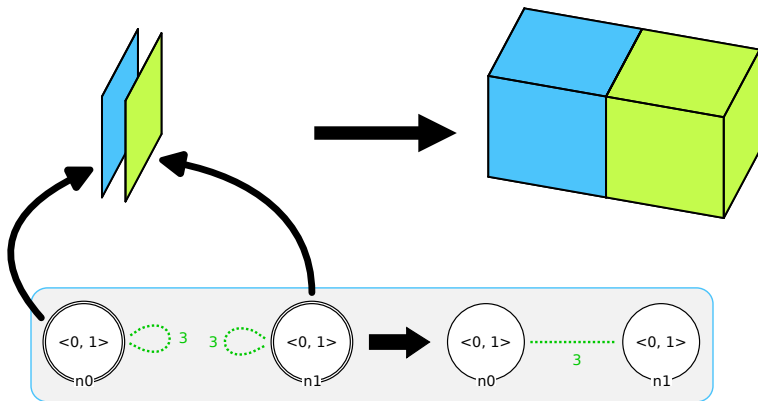
¹Poudret et al. 2008.

²Belhaouari et al. 2014.

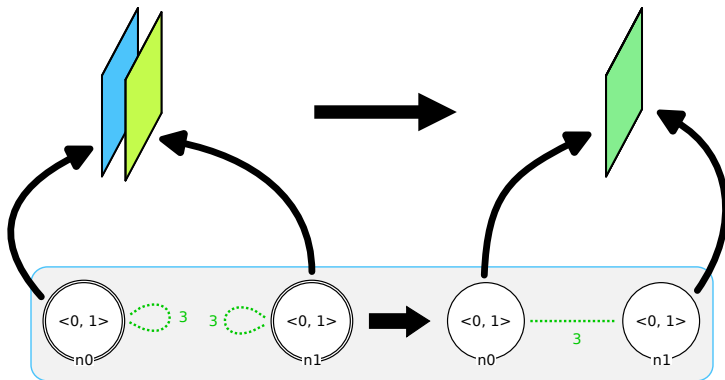
A sneak peek at Jerboa's language



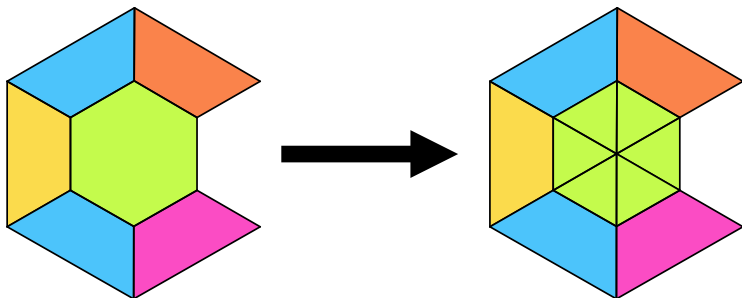
A sneak peek at Jerboa's language



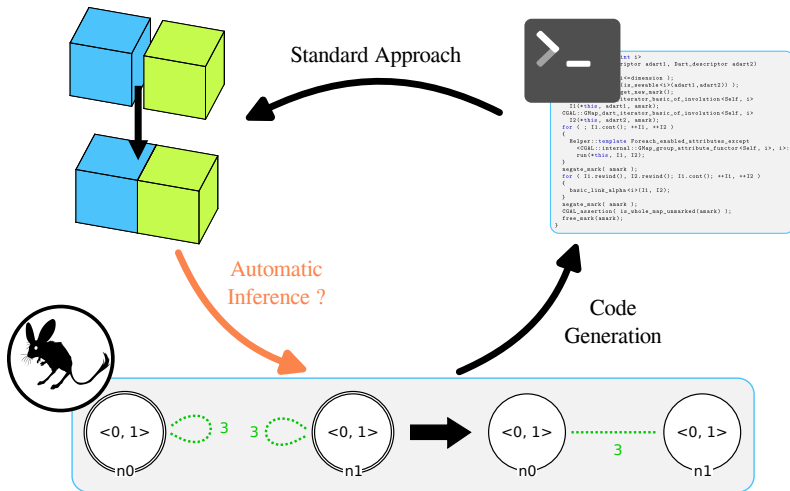
A sneak peek at Jerboa's language



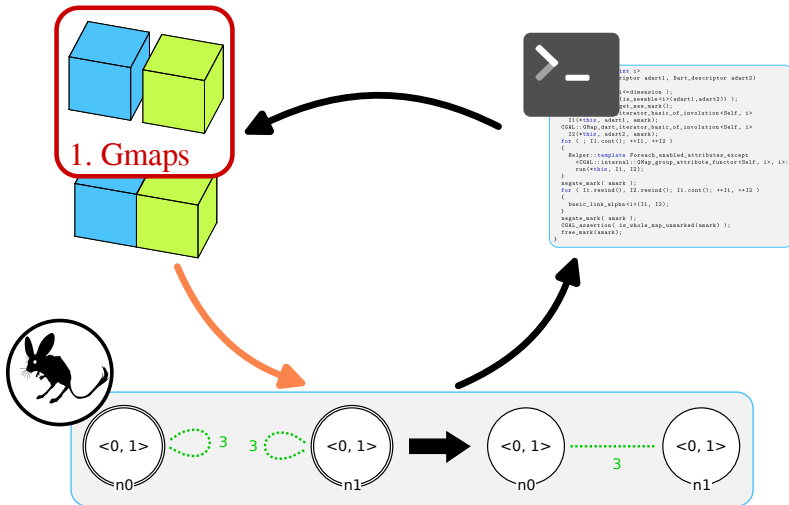
Running example: face triangulation



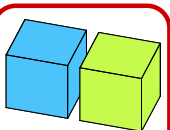
Plan



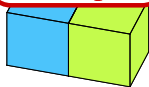
Plan



Plan

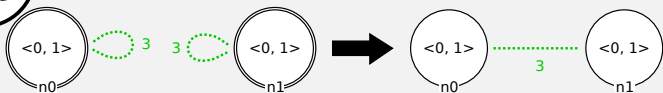


1. Gmaps



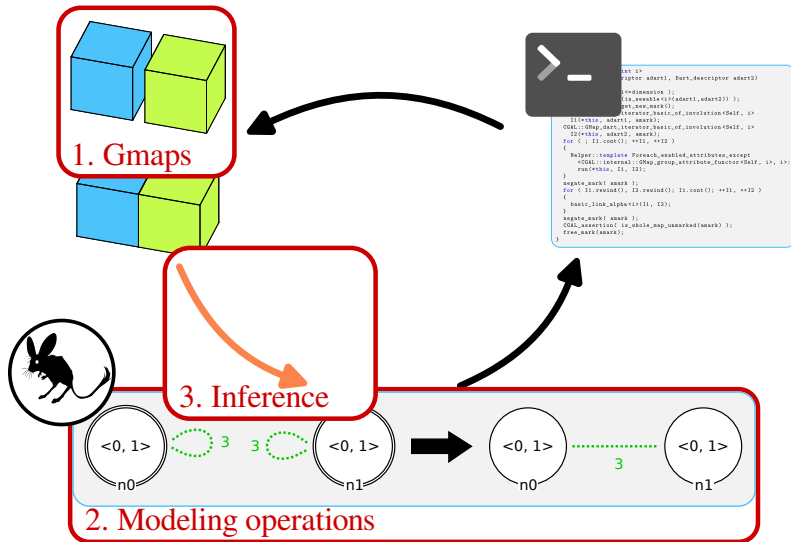
```

    int i>
    descriptor adart1, Dart_descriptor adart2)
    {
        i<<dimension();
        if (is_sevable<i>(adart1,adart2)) );
        get_new_mark();
        Iterator::Basic_of_Involution<Self, i>
        II(*this, adart1, mark);
        CGM::CGM_dart_iterator::Basic_of_Involution<Self, i>
        I2(*this, adart2, mark);
        for ( ; I1.cont(); ++I1, ++I2 )
        {
            Helper::template ForEach_enabled_attributes_except
            <CGM::Interval::OMap_group_attributes_function<Self, i>, i>;
            run(*this, I1, I2);
        }
        segregate_mark( mark );
        for ( I1.reset(); I1.cont(); ++I1, ++I2 )
        {
            basic_link_alpha<i>(I1, I2);
        }
        segregate_mark( mark );
        CGM::assertion( is_whole_map_unmarked(mark) );
        free_mark(mark);
    }
  
```

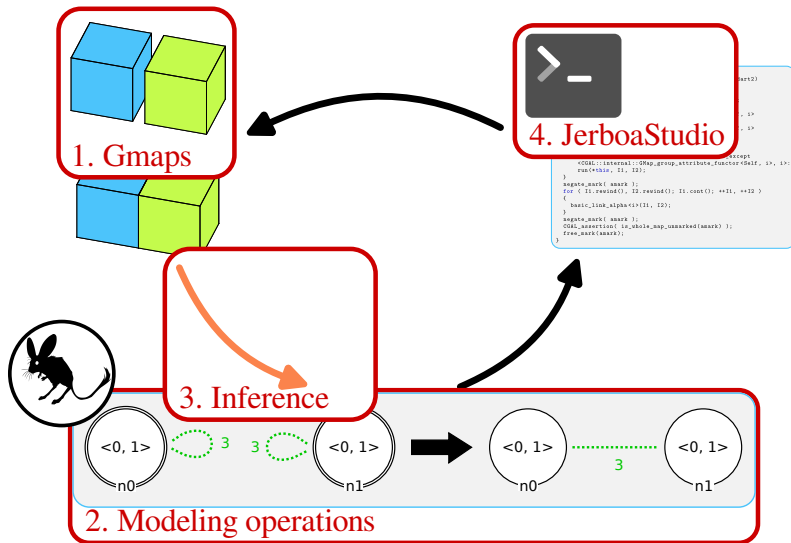


2. Modeling operations

Plan



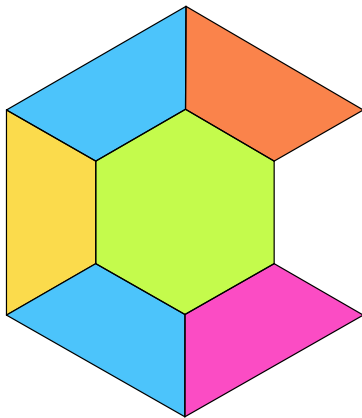
Plan



Generalized maps

- ▶ Geometric objects are represented with embedded generalized maps.

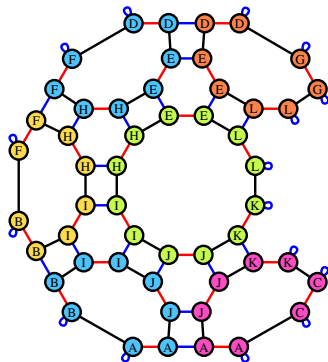
Generalized maps¹



¹Damiand et al. 2014.



Generalized maps¹



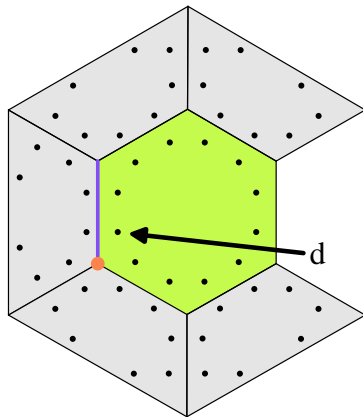
Gmaps built as graphs.

Color legend: 0, 1, 2.

¹Damiand et al. 2014.



Generalized maps¹



Gmaps built as graphs.

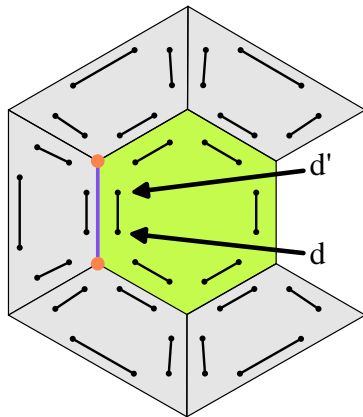
d identifies the orange vertex, the purple edge, and the green face.

Color legend: 0, 1, 2.

¹Damiand et al. 2014.



Generalized maps¹



Gmaps built as graphs.

0-arc:

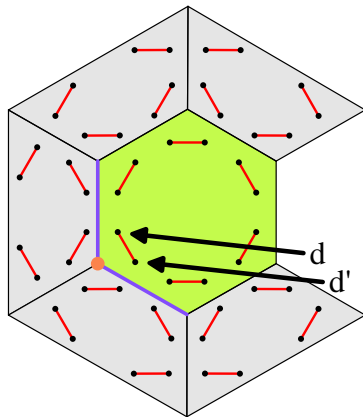
- distinct vertices.
- same edge and faces.

Color legend: 0, 1, 2.

¹Damiand et al. 2014.



Generalized maps¹



Gmaps built as graphs.

1-arc:

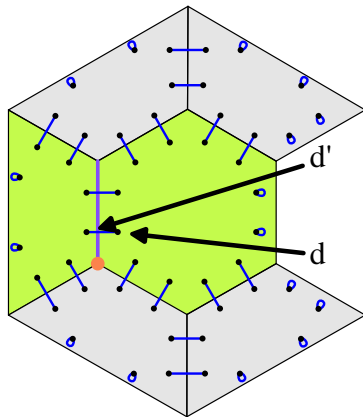
- distinct edges.
- same vertices and faces.

Color legend: 0, 1, 2.

¹Damiand et al. 2014.



Generalized maps¹



2-arc:

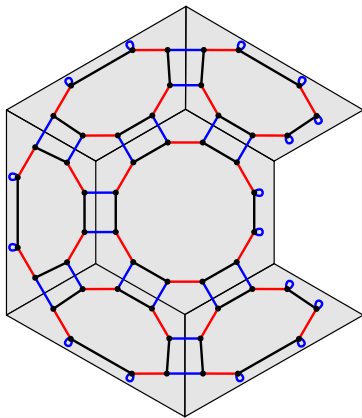
- distinct faces.
- same vertices and edges.

Color legend: 0, 1, 2.

¹Damiand et al. 2014.



Generalized maps¹



Gmaps built as graphs.

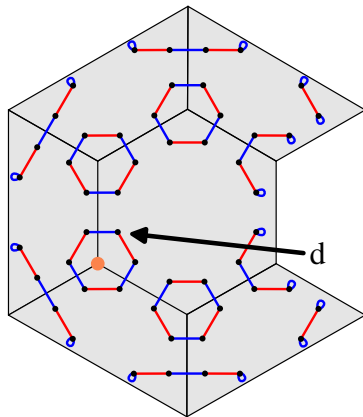
Full topology obtained by
superimposing the relations.

Color legend: 0, 1, 2.

¹Damiand et al. 2014.



Orbits and topological cells



► Orbit (encode topological cell):

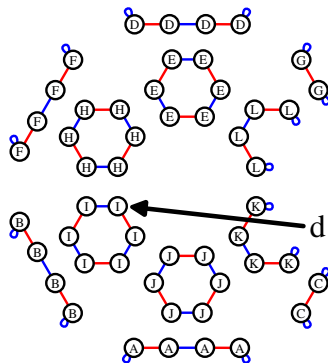
Graph induced by a subset $\langle o \rangle \subseteq \llbracket 0, n \rrbracket$ of dimensions.

- Vertices: orbits $\langle 1, 2 \rangle$.

Color legend: 0, 1, 2.



Orbits and topological cells



► Orbit (encode topological cell):

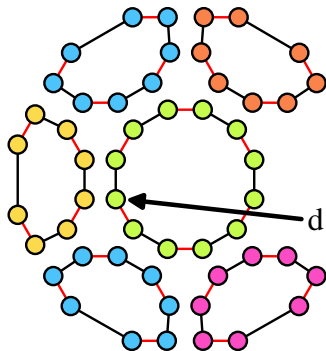
Graph induced by a subset $\langle o \rangle \subseteq \llbracket 0, n \rrbracket$ of dimensions.

- Vertices: orbits $\langle 1, 2 \rangle$.
- Carry positions.

Color legend: 0, 1, 2.



Orbits and topological cells



► Orbit (encode topological cell):

Graph induced by a subset $\langle o \rangle \subseteq \llbracket 0, n \rrbracket$ of dimensions.

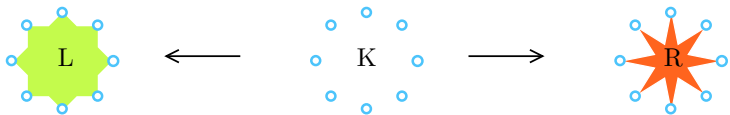
- Vertices: orbits $\langle 1, 2 \rangle$.
- Faces: orbits $\langle 0, 1 \rangle$.
- Carry colors.

Color legend: 0, 1, 2.

Formalizing modeling operations

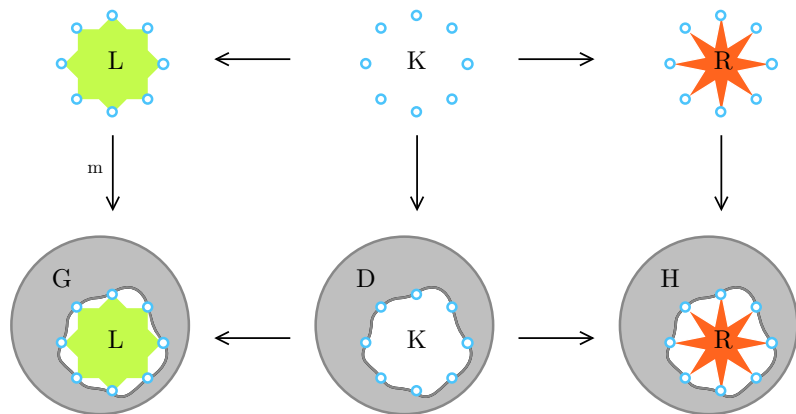
- ▶ Operations on Gmaps are designed as graph rewriting rules.

Graph transformation rules¹



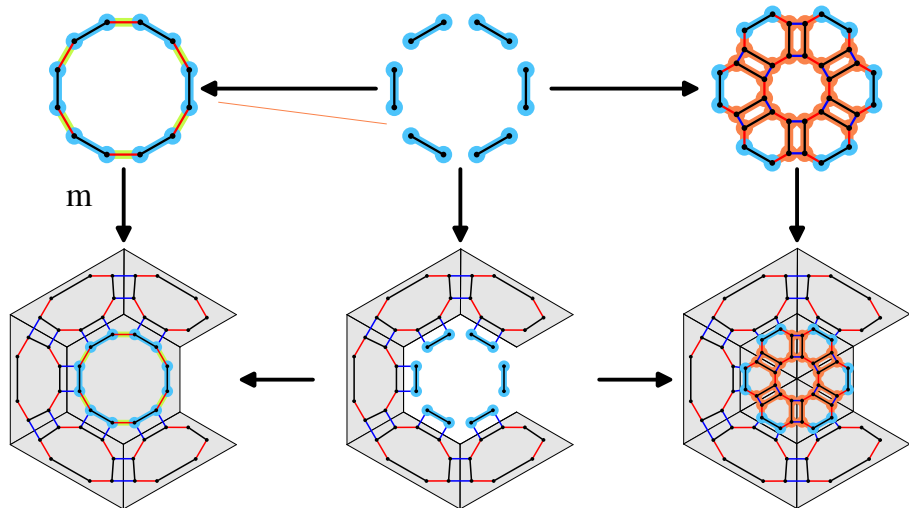
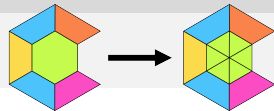
¹Rozenberg 1997; Ehrig et al. 2006; Heckel et al. 2020.

Graph transformation rules¹

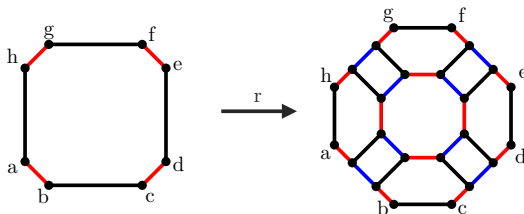


¹Rozenberg 1997; Ehrig et al. 2006; Heckel et al. 2020.

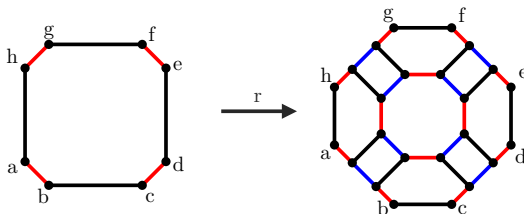
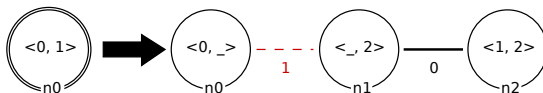
Rewriting Gmaps



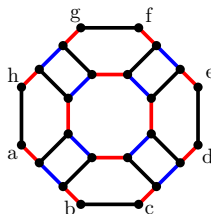
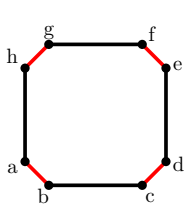
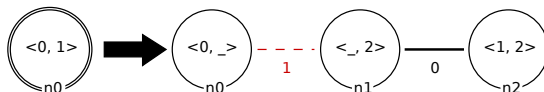
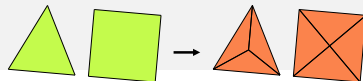
Orbit rewriting



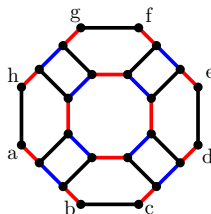
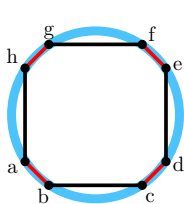
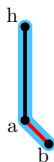
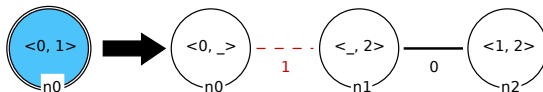
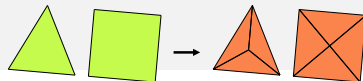
Orbit rewriting



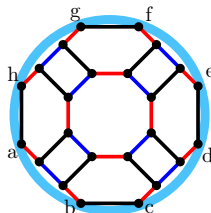
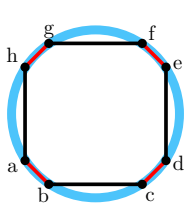
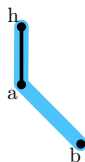
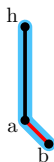
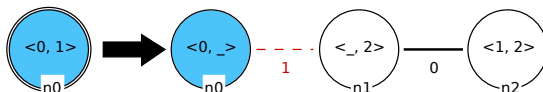
Orbit rewriting



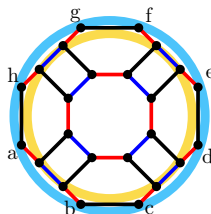
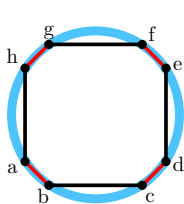
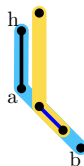
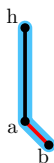
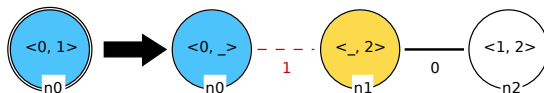
Orbit rewriting



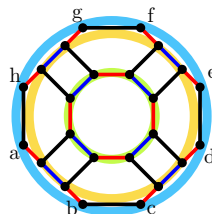
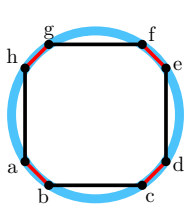
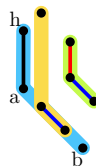
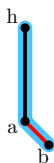
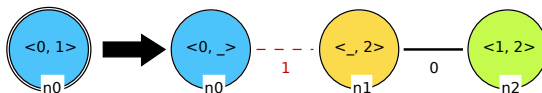
Orbit rewriting



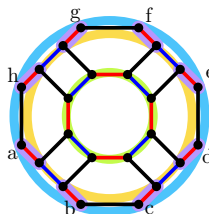
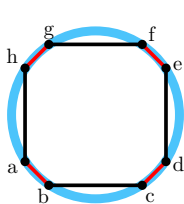
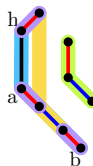
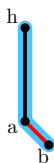
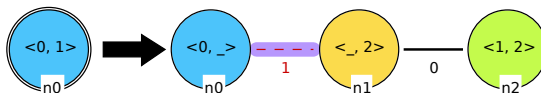
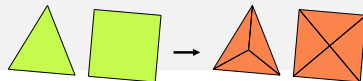
Orbit rewriting



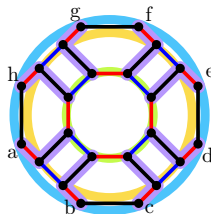
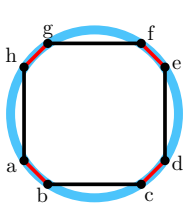
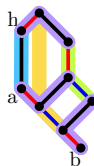
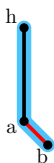
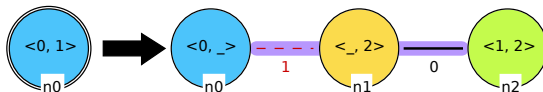
Orbit rewriting



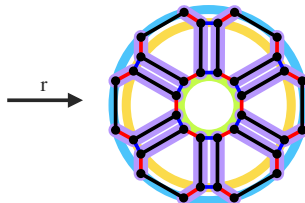
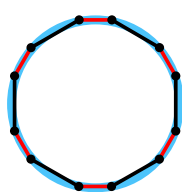
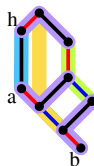
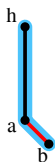
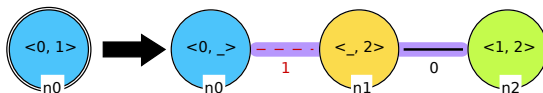
Orbit rewriting



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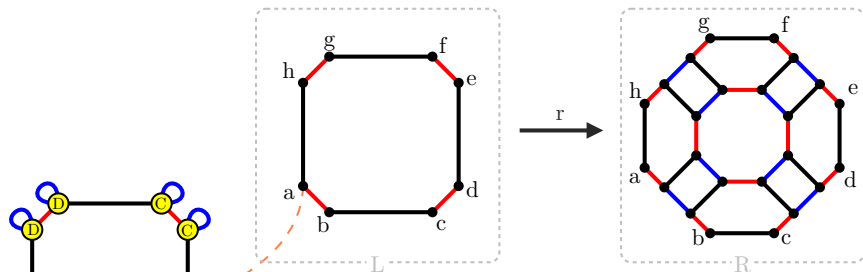
Orbit rewriting



Modifying geometric values¹

¹Bellet et al. 2017.

Modifying geometric values¹

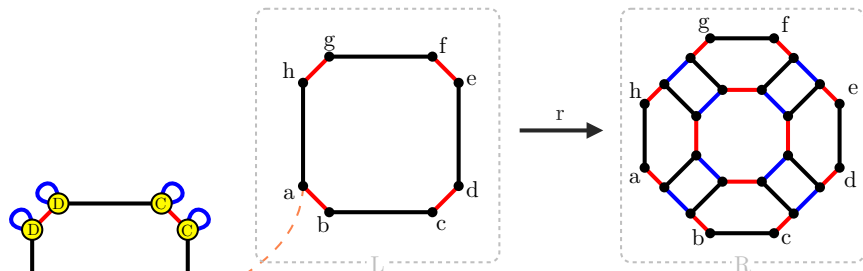


Embedding expressions modeled with algebraic data types:

- add
- middle
- scale
- ...

¹Bellet et al. 2017.

Modifying geometric values¹

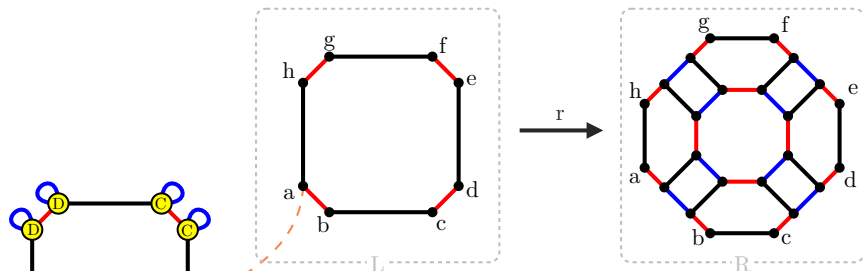


Embedding expressions are extended with topological operators:

- Neighbor operator:
 - ▶ $a@0@1@0.position = f.position = C$
 - ▶ $a@1@0.color = c.color = \text{yellow}$

¹Bellet et al. 2017.

Modifying geometric values¹

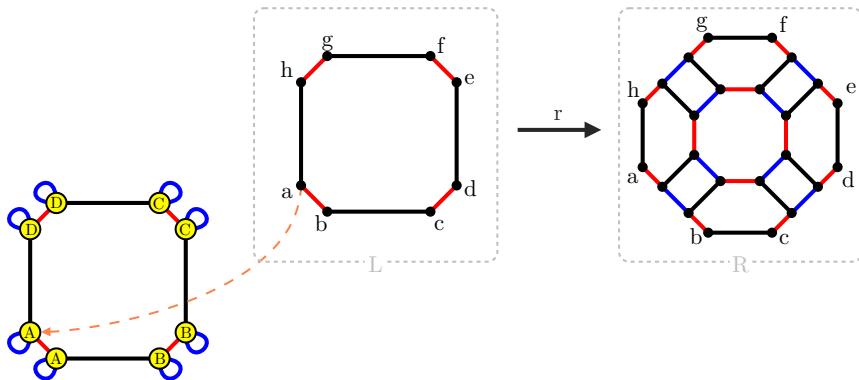
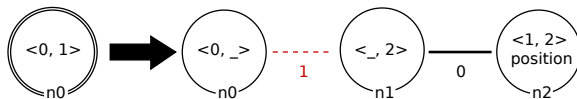


Embedding expressions are extended with topological operators:

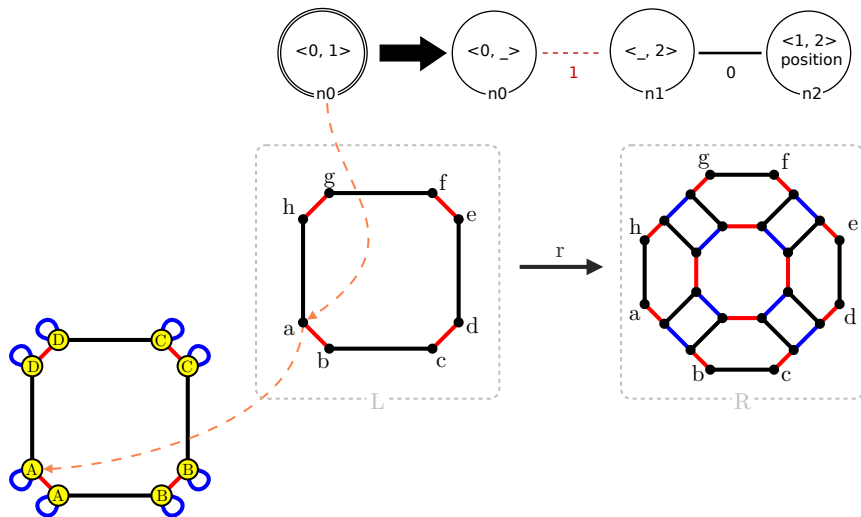
- Neighbor operator:
- Collect operator:
 - ▶ $position_{\langle 0,1 \rangle}(a) = \{A, B, C, D\}$
 - ▶ $color_{\langle 0,1 \rangle}(a) = \{\text{yellow}\}$

¹Bellet et al. 2017.

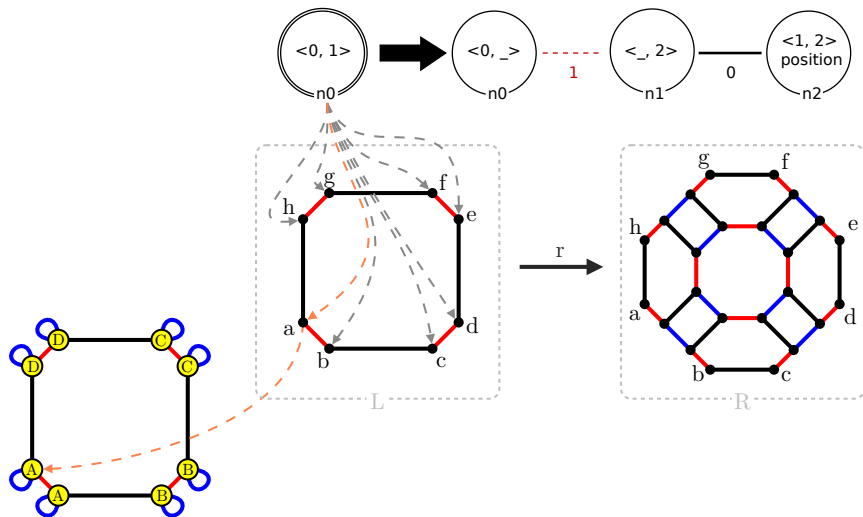
Extension to schemes



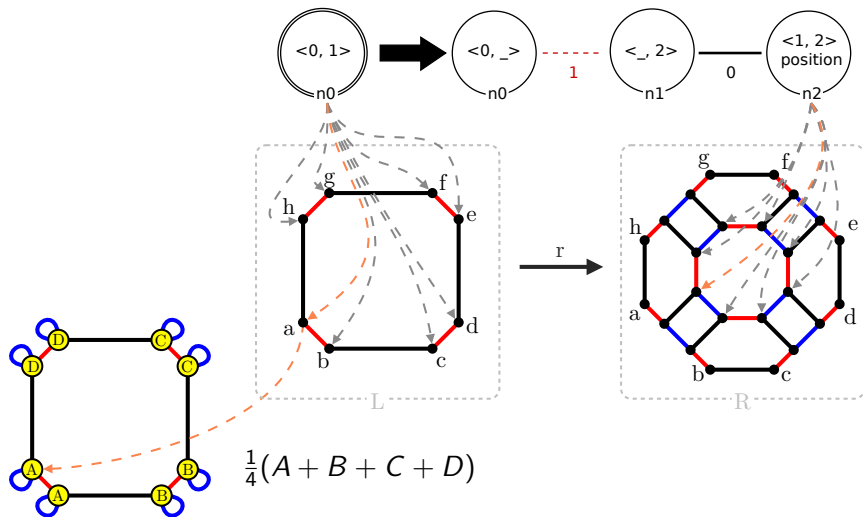
Extension to schemes



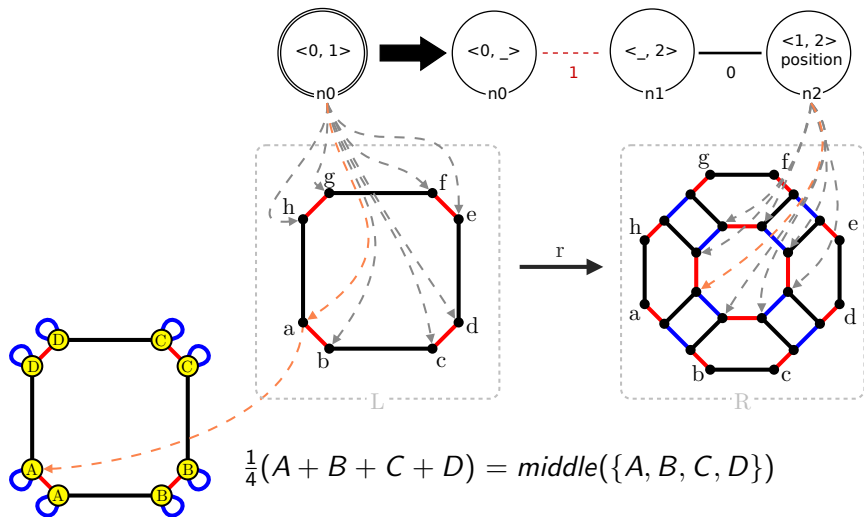
Extension to schemes



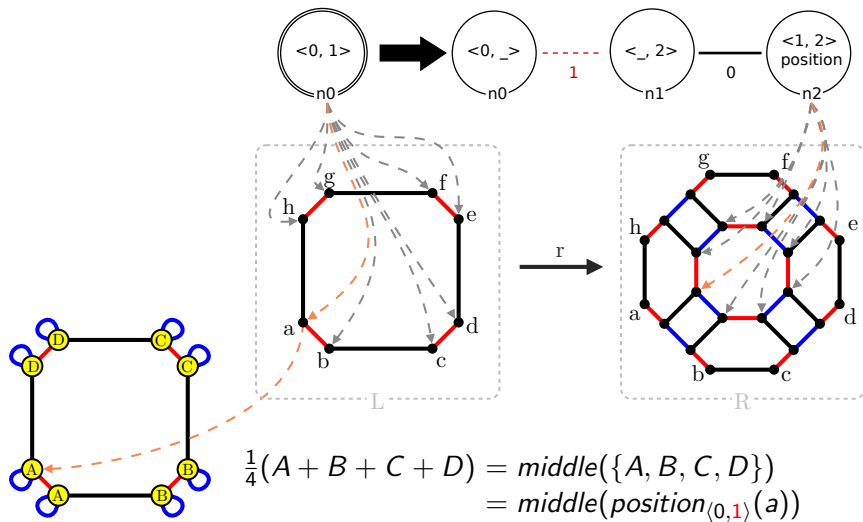
Extension to schemes



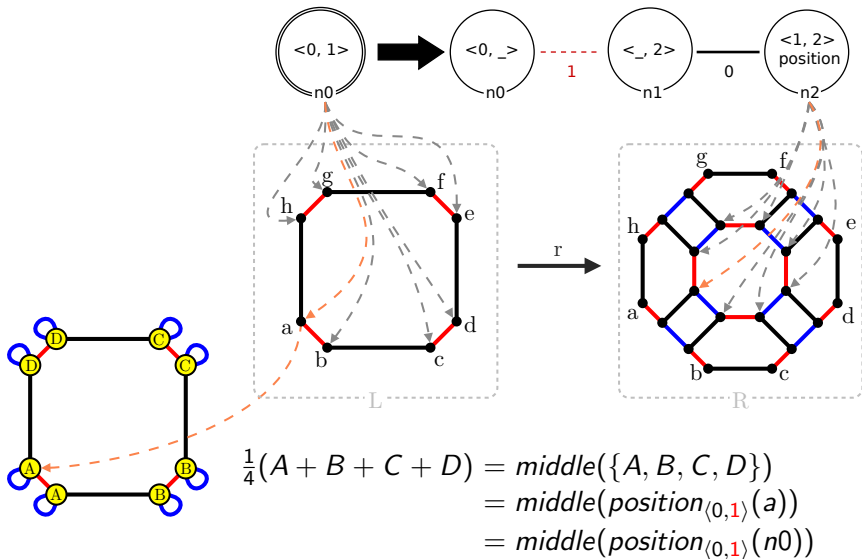
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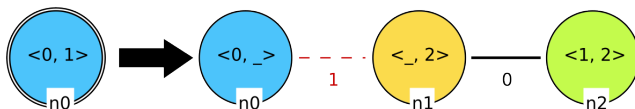


Extension to schemes



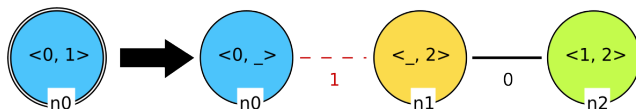
Key points

We describe geometric modeling operations with rule schemes.



Key points

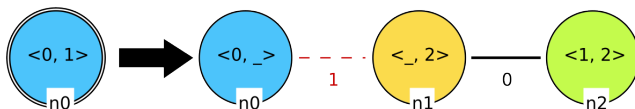
We describe geometric modeling operations with rule schemes.



- Operations described in a domain-specific language

Key points

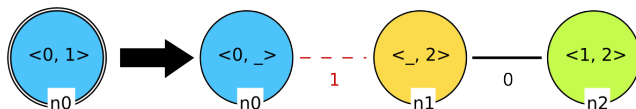
We describe geometric modeling operations with rule schemes.



- Operations described in a domain-specific language
- Each node in the rule encodes for multiple darts in the Gmaps.

Key points

We describe geometric modeling operations with rule schemes.

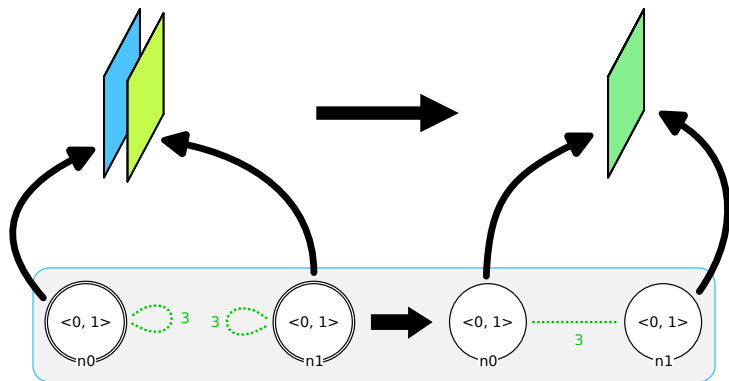


- Operations described in a domain-specific language
- Each node in the rule encodes for multiple darts in the Gmaps.
- Node names substituted by dart names during the instantiation.

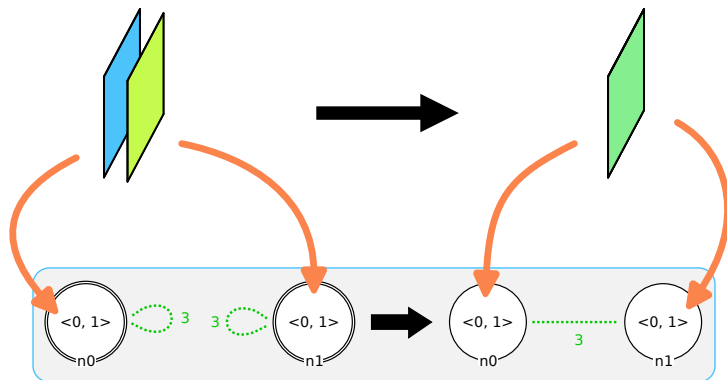
Inferring geometric modeling operations

- ▶ Retrieving the operation described by an example.

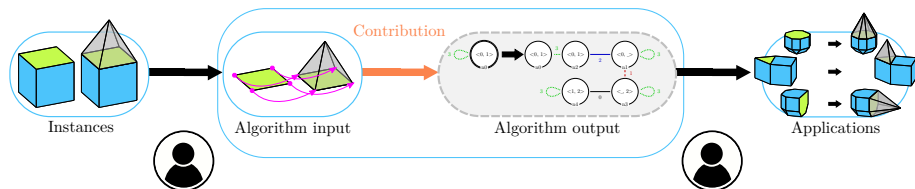
Reversing the instantiation process



Reversing the instantiation process

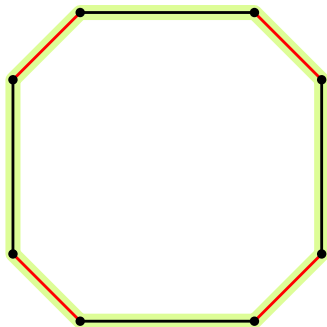


Inference workflow

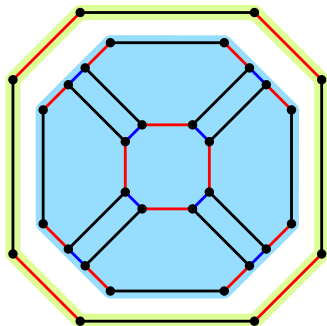


- **Input:** A graph G encoding the preservation relation between two partial Gmaps, an orbit type $\langle o \rangle$ and a dart a of G .
- **Output:** A graph S that encodes the Jerboa rule with the variable $\langle o \rangle$, given that the operation is applied to the dart a .

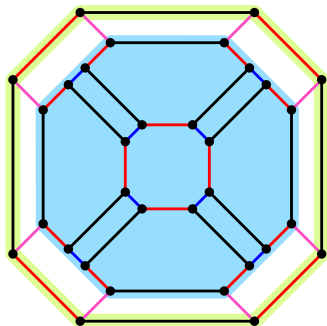
Folding a joint representation of the rule



Folding a joint representation of the rule

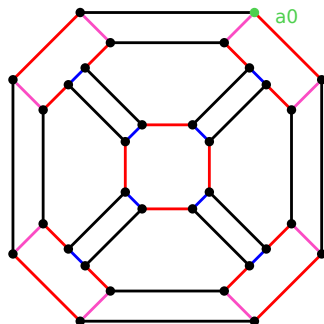


Folding a joint representation of the rule



Color legend: 0, 1, 2, κ .

Folding a joint representation of the rule



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Besides the two Gmaps and the preservation links, we chose a **dart** in the initial Gmap and an **orbit type**.

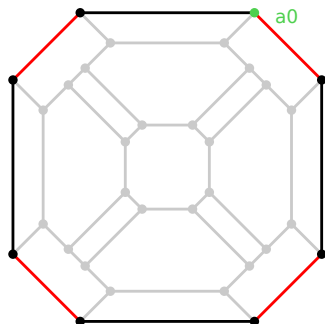
► Graph traversal algorithm.

Iteratively applying two foldings:

- Folding of a node.
- Folding of the arcs.

► Illustration on face triangulation with the orbit type $\langle 0, 1 \rangle$ and the dart **a0**.

Execution

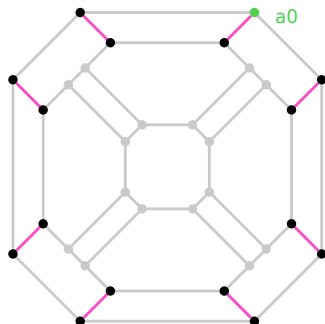


Creation of the hook (orbit $\langle 0, \mathbf{1} \rangle$).



Color legend: 0, $\mathbf{1}$, $\mathbf{2}$, κ .

Execution

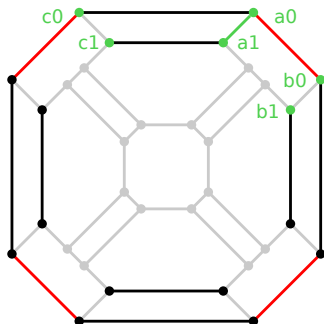


Folding of the arcs.



Color legend: 0, 1, 2, K .

Execution

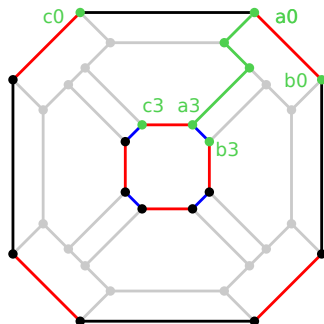


Folding of a node.



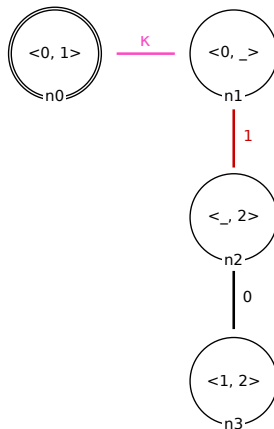
Color legend: 0, 1, 2, κ .

Execution

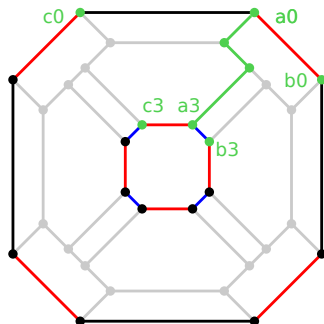


Color legend: 0, 1, 2, κ .

The algorithm terminates.

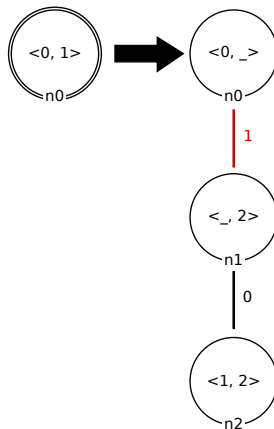


Execution



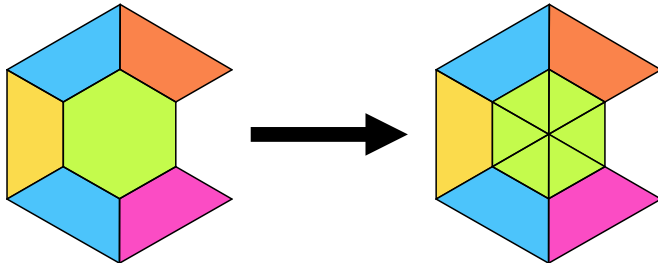
Color legend: 0, 1, 2, κ .

Splitting the joint representation.



Results¹

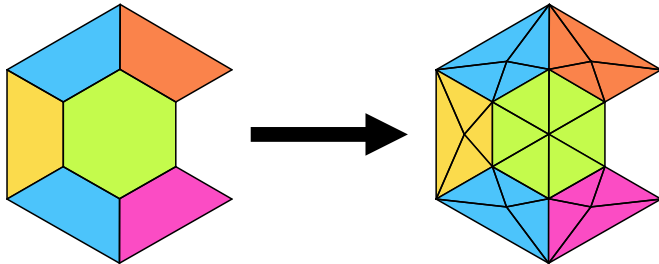
- **Correctness:** The algorithm returns a topological folding of the rule if it exists and halts otherwise.
- What about cases where we cannot fold the rule? Example with the orbit $\langle 0, 1, 2 \rangle$.



¹Pascual et al. 2022a.

Results¹

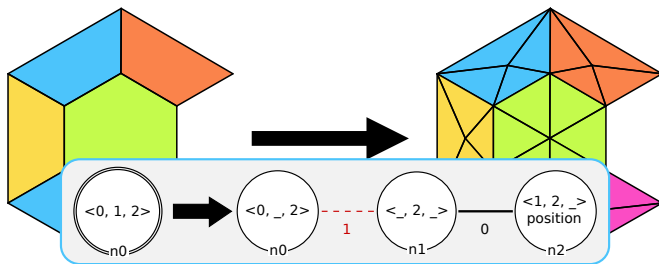
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Method for the geometric modifications (positions)

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- t : translation (unknown)

Need for abstraction on schemes

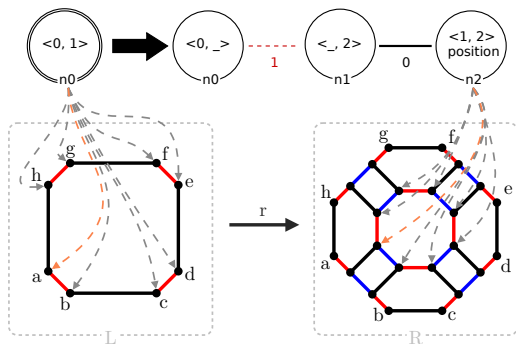
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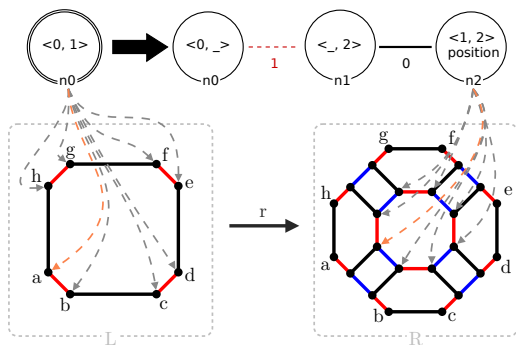
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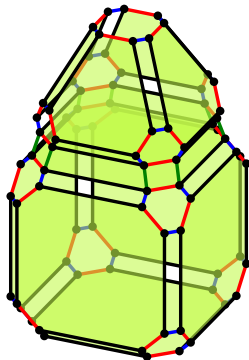
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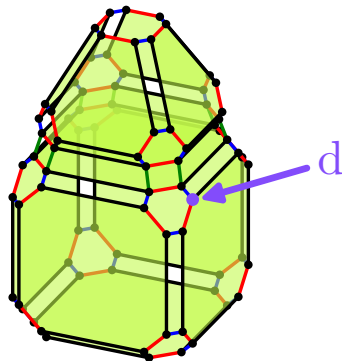
Solution: Exploit the topology.

► Use points of interest that share the same expression.

Points of interest



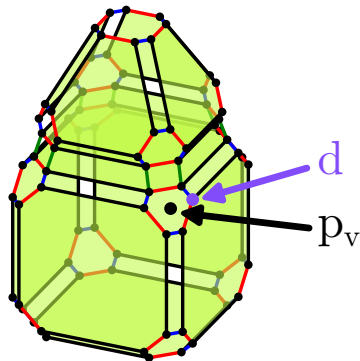
Points of interest



Points of interest

with

- p_v : vertex

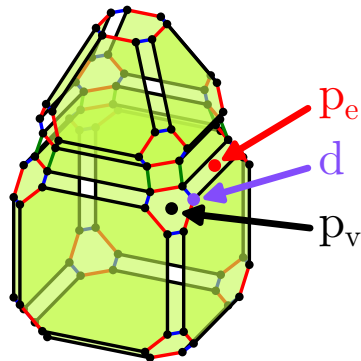


$$p_v = \text{middle}(\text{position}_{\langle 1,2,3 \rangle}(d))$$

Points of interest

with

- p_v : vertex
- p_e : edge midpoint

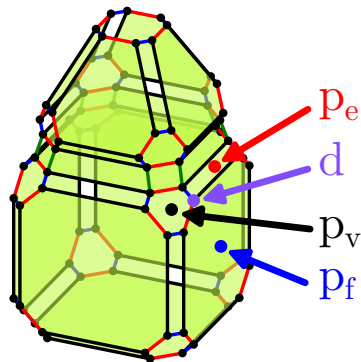


$$p_e = \text{middle}(\text{position}_{\langle 0,2,3 \rangle}(d))$$

Points of interest

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter

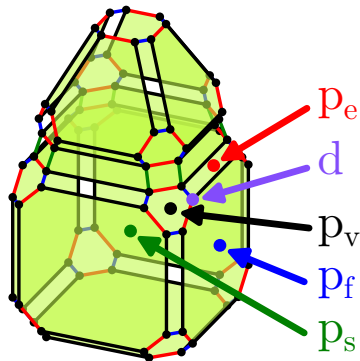


$$p_f = \text{middle}(\text{position}_{\langle 0,1,3 \rangle}(d))$$

Points of interest

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter
- p_s : volume barycenter

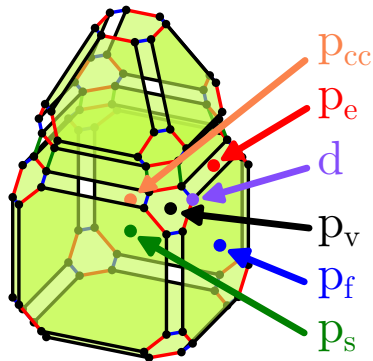


$$p_s = \text{middle}(\text{position}_{\langle 0,1,2 \rangle}(d))$$

Points of interest

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter
- p_s : volume barycenter
- p_{cc} : CC barycenter

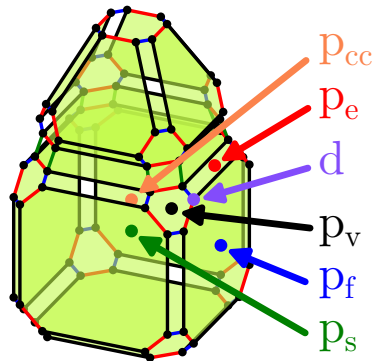


$$p_{cc} = \text{middle}(\text{position}_{\langle 0,1,2,3 \rangle}(d))$$

Points of interest

with

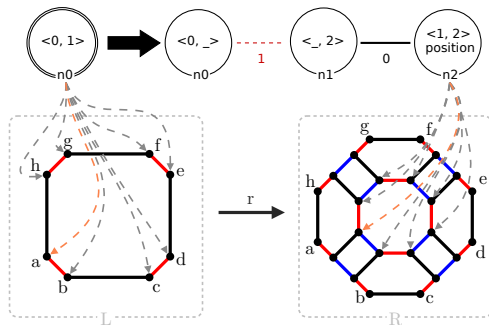
- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter
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Thanks to the points of interest, the system is rewritten as:

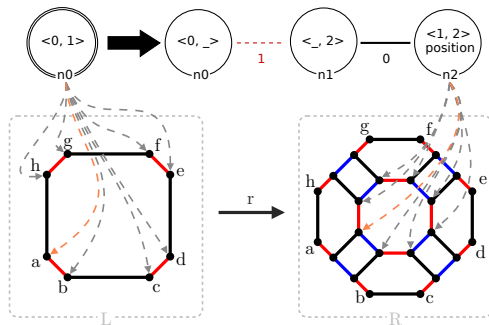
$$p = w_v p_v + w_e p_e + w_f p_f + w_s p_s + w_{cc} p_{cc} + t$$

Illustration



The position expression of n_2 only depends on n_0 .

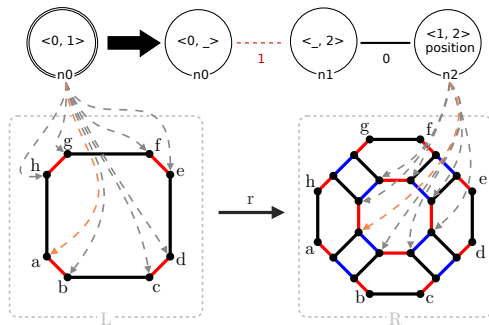
Illustration



The position expression of $n2$ only depends on $n0$.

$$n2.position = \underbrace{w_v n0.p_v}_{\text{vertex}} + \underbrace{w_e n0.p_e}_{\text{edge}} + \underbrace{w_f n0.p_f}_{\text{face}} + \underbrace{w_s n0.p_s}_{\text{volume}} + \underbrace{w_{cc} n0.p_{cc}}_{\text{cc}} + t$$

Illustration

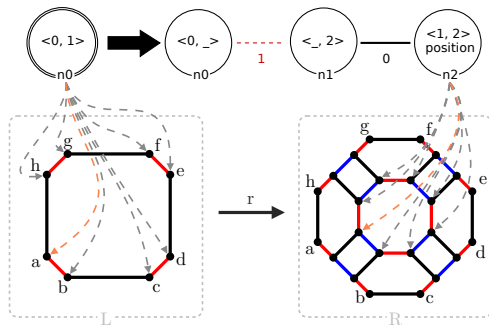


The position expression of $n2$ only depends on $n0$.

- One equation per dart (8 darts).

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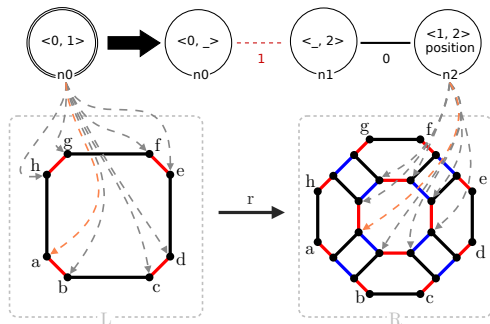


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Illustration

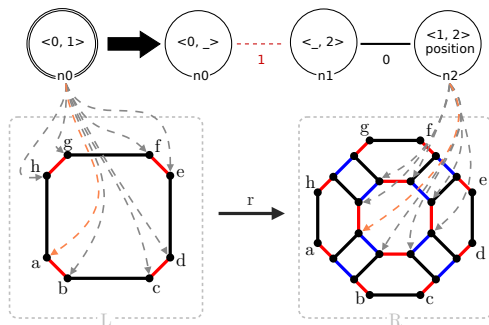


The position expression of n_2 only depends on n_0 .

- One equation per dart (8 darts).
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- 24 equations and 8 variables.

$$n_2.position = \underbrace{w_v n_0.p_v}_{\text{vertex}} + \underbrace{w_e n_0.p_e}_{\text{edge}} + \underbrace{w_f n_0.p_f}_{\text{face}} + \underbrace{w_s n_0.p_s}_{\text{volume}} + \underbrace{w_{cc} n_0.p_{cc}}_{\text{cc}} + t$$

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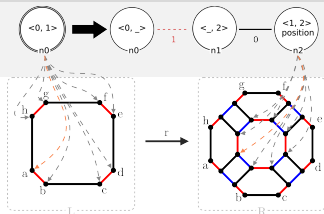
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► Solved as a CSP. Solvers used: OR-Tools (Google), Z3 (Microsoft)

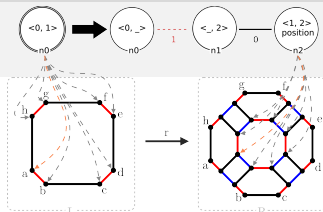
Solving the barycentric triangulation

► Global equation:

$$n2.position = w_v n0.p_v + w_e n0.p_e + w_f n0.p_f + w_s n0.p_s + w_{cc} n0.p_{cc} + t$$



Solving the barycentric triangulation



► Global equation:

$$n2.position = w_v n0.p_v + w_e n0.p_e + w_f n0.p_f + w_s n0.p_s + w_{cc} n0.p_{cc} + t$$

► Generated system (only on x and y)

$$\begin{cases} (0.5; 0.5) = w_v * (0; 0) + w_e * (0.5; 0) + w_f * (0.5; 0.5) + w_s * (0.5; 0.5) + w_{cc} * (0.5; 0.5) + (tx; ty) \\ (0.5; 0.5) = w_v * (1; 0) + w_e * (0.5; 0) + w_f * (0.5; 0.5) + w_s * (0.5; 0.5) + w_{cc} * (0.5; 0.5) + (tx; ty) \\ (0.5; 0.5) = w_v * (1; 0) + w_e * (1; 0.5) + w_f * (0.5; 0.5) + w_s * (0.5; 0.5) + w_{cc} * (0.5; 0.5) + (tx; ty) \\ (0.5; 0.5) = w_v * (1; 1) + w_e * (1; 0.5) + w_f * (0.5; 0.5) + w_s * (0.5; 0.5) + w_{cc} * (0.5; 0.5) + (tx; ty) \\ \vdots \\ \vdots \end{cases}$$

- ▶ Generated system (only on x and y)

$$n2.position = w_v n0.p_v + w_e n0.p_e + w_f n0.p_f + w_s n0.p_s + w_{cc} n0.p_{cc} + t$$

[illegible]

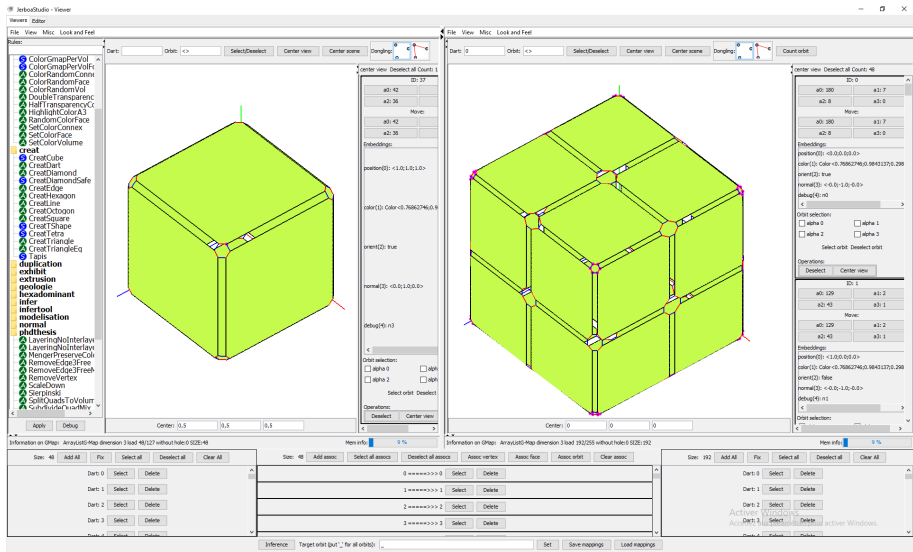
- Solution found:

- $w_v = 0.0$
- $w_e = 0.0$
- $w_f = 1.0$
- $w_s = 0.0$
- $w_{cc} = 0.0$
- $t = (0.0, 0.0)$

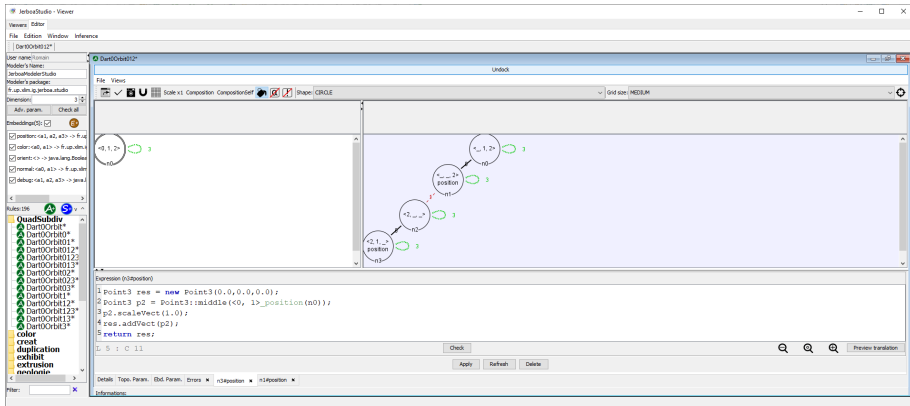
JerboaStudio and applications

- ▶ Implementation of the inference mechanism in Jerboa.

JerboaStudio: inferring the quad subdivision



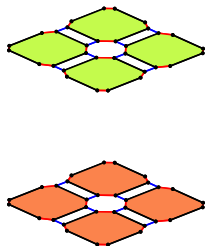
Folding the quad subdivision



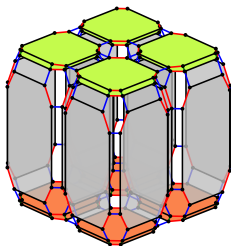
- 768 possible schemes
- 48 schemes tried (marking).
- 14 schemes built (removal of isomorphic rules).

Example inspired from geology

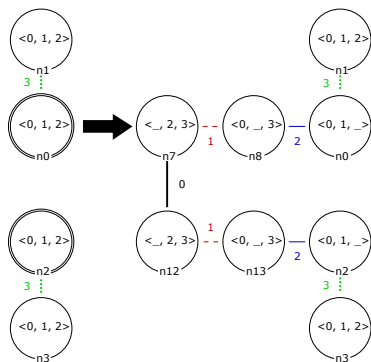
Before



After



Operation



Inference time: ~ 3 ms

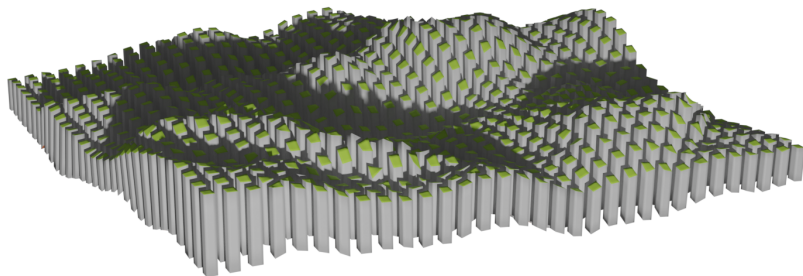
Example inspired from geology

Before



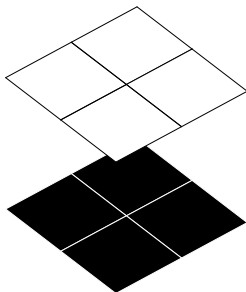
Example inspired from geology

After

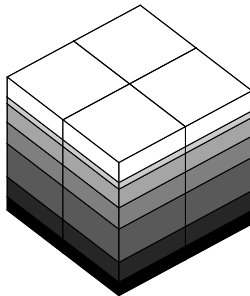


Example inspired from geology (part 2)

Before



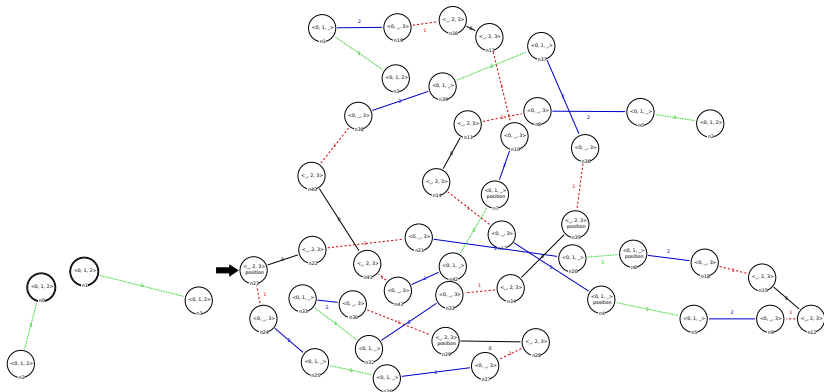
After



- We infer interpolations both for the positions and the colors.

Example inspired from geology (part 2)

Operation



Inference time: ~ 26 ms for the topology,
 ~ 549 ms for the embedding expressions

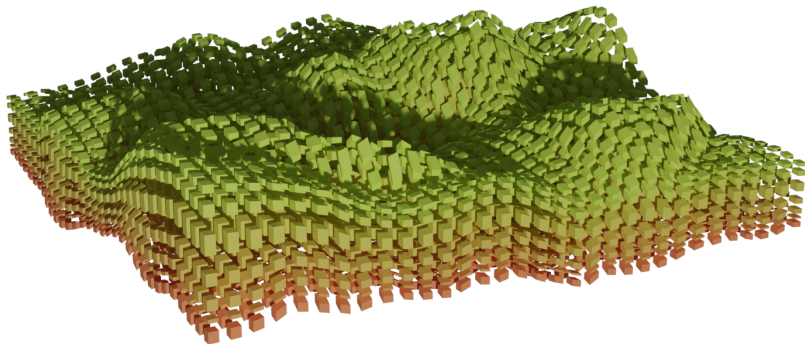
Example inspired from geology (part 2)

Before



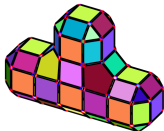
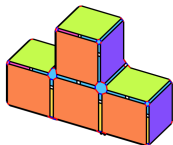
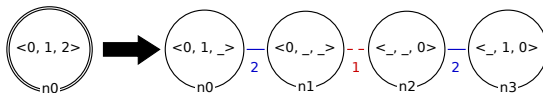
Example inspired from geology (part 2)

After



Doo-Sabin subdivision¹

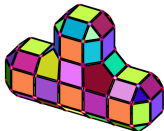
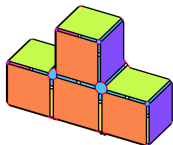
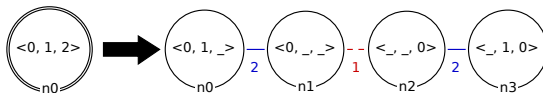
► Rule scheme used and inferred:



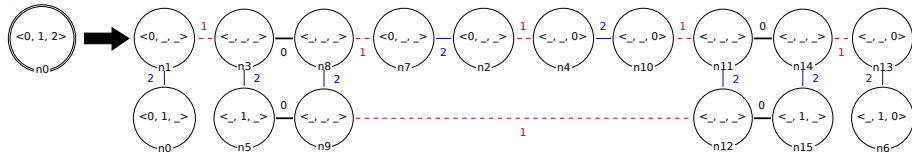
¹Doo et al. 1978.

Doo-Sabin subdivision¹

► Rule scheme used and inferred:



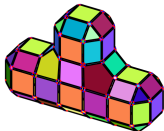
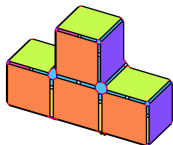
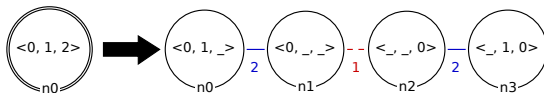
► 2nd iteration:



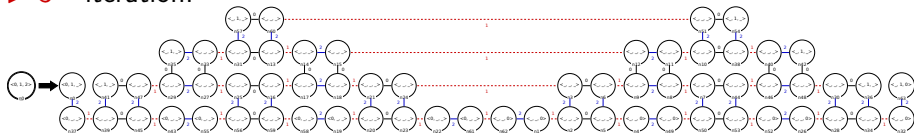
¹Doo et al. 1978.

Doo-Sabin subdivision¹

► Rule scheme used and inferred:

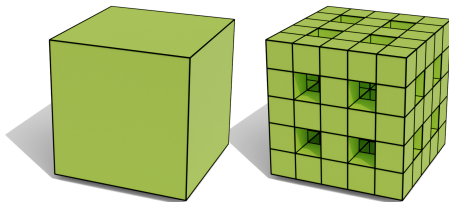


► 3rd iteration:



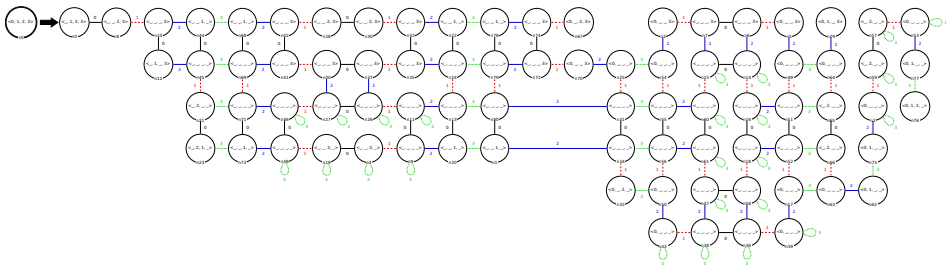
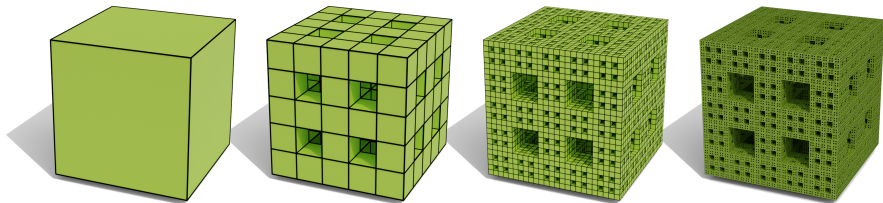
¹Doo et al. 1978.

Menger $(2, 2, 2)^1$



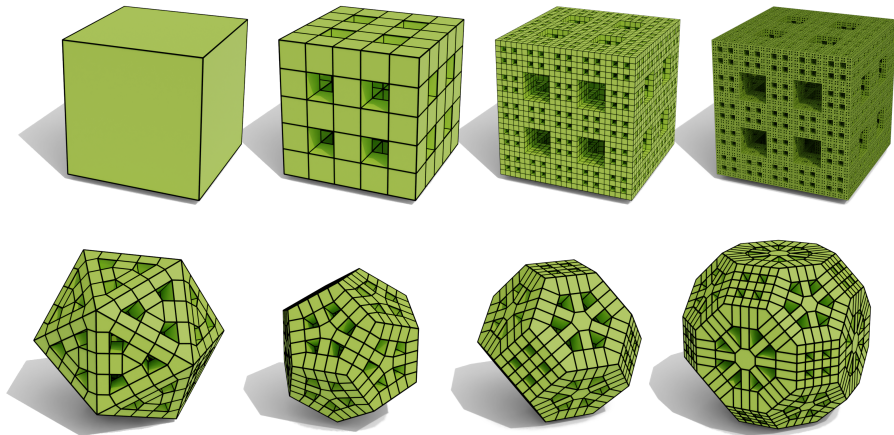
¹Richaume et al. 2019.

Menger $(2, 2, 2)^1$



¹Richaume et al. 2019.

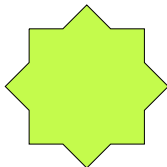
Menger $(2, 2, 2)^1$



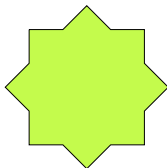
¹Richaume et al. 2019.

Edge cases

- ▶ Von Koch's snowflake generated with L-systems

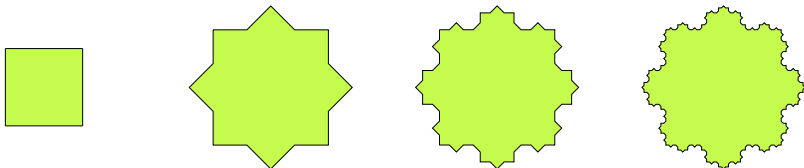


- ▶ Inferred:

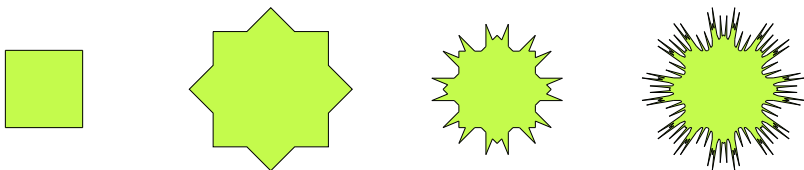


Edge cases

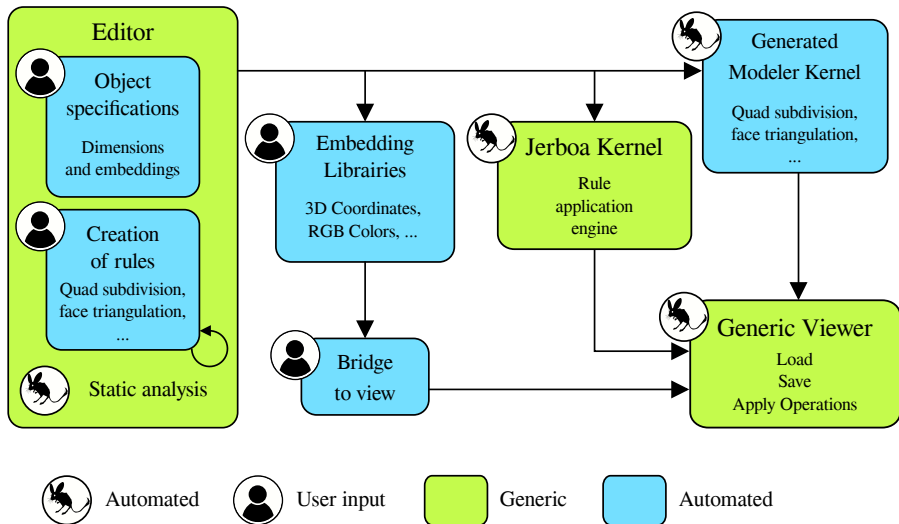
- ▶ Von Koch's snowflake generated with L-systems



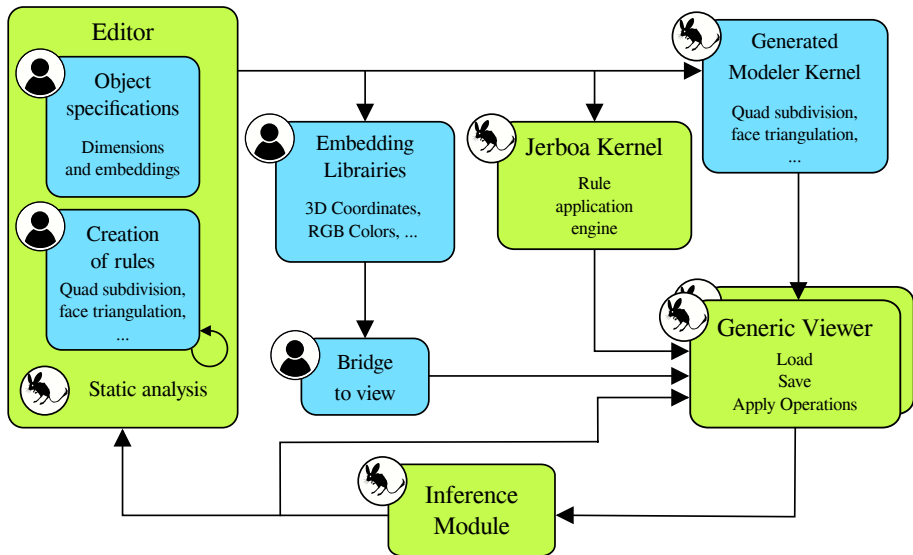
- ▶ Inferred:



JerboaStudio's architecture



JerboaStudio's architecture



Conclusion

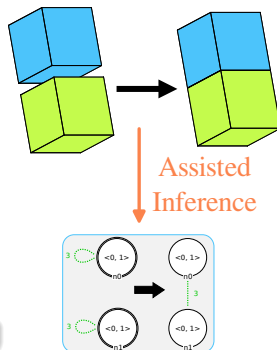
- ▶ Current research topics.

Main contributions

► Inference of modeling operations:

- Topological folding algorithm¹
- Values of interest and CSP

► JerboaStudio.



► Graph transformations for geometric modeling:

- Graph products²
- Rule completion³

► Unified framework for combinatorial maps.

¹Pascual et al. 2022a

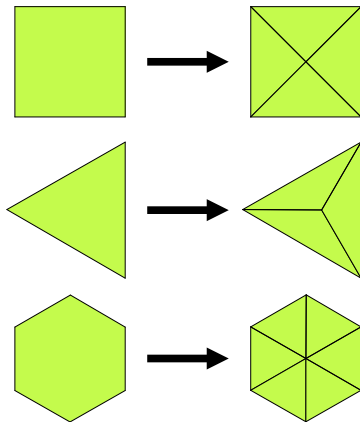
²Pascual et al. 2022b

³Arnould et al. 2022

Future works

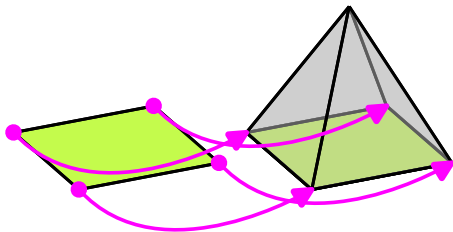
► Instance generation

- Create objects on which a given rule scheme is applicable.



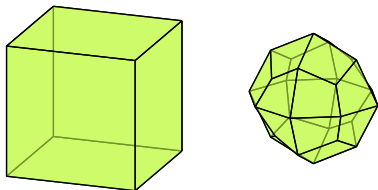
Future works

- ▶ Instance generation
- ▶ Automatic mapping
 - Cumbersome step in the inference workflow.



Future works

- ▶ Instance generation
- ▶ Automatic mapping
- ▶ Other hypotheses for the geometric inference
 - Most subdivision schemes rely on other computations: the Catmull-Clark subdivision.¹



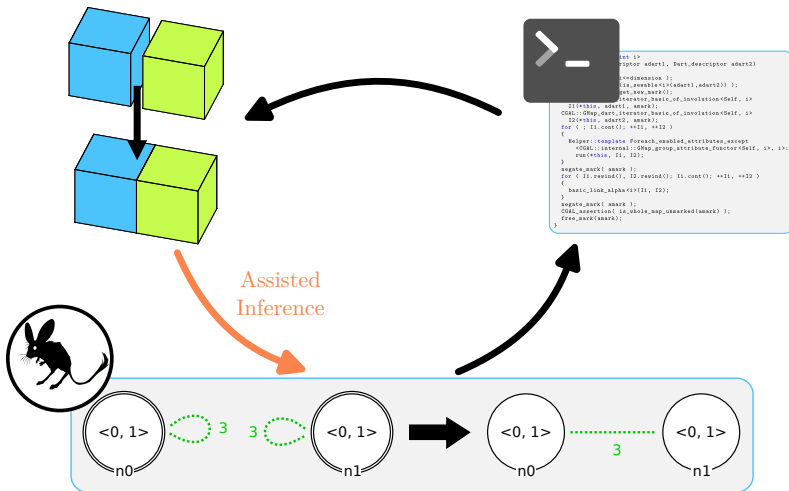
```
// From Catmull and Clark 1978, Recursively generated
// B-spline surfaces on arbitrary topological meshes

// n1#position
// midpoint of the incident face
Point3 face1Mid = Point3::middle(<0,1>_position(n0));
// midpoint of the adjacent face
Point3 face2Mid = Point3::middle(<0,1>_position(n0@2));
// average of the face points
Point3 faceMid = Point3::middle(face1Mid, face2Mid);
// midpoint of the edge
Point3 edgeMid = Point3::middle(<0>_position(n0));
// average of the edge and face points
return Point3::middle(faceMid, edgeMid);
```

Out of scope

¹Catmull et al. 1978.

Thank you for listening



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-  Št'ava, Ondrej et al. (2010). “Inverse Procedural Modeling by Automatic Generation of L-systems”. In: **Computer Graphics Forum** 29.2, pp. 665–674. ISSN: 1467-8659. DOI: [10.1111/j.1467-8659.2009.01636.x](https://doi.org/10.1111/j.1467-8659.2009.01636.x).
-  Wu, Fuzhang et al. (July 27, 2014). “Inverse procedural modeling of facade layouts”. In: **ACM Transactions on Graphics** 33.4, 121:1–121:10. ISSN: 0730-0301. DOI: [10.1145/2601097.2601162](https://doi.org/10.1145/2601097.2601162).

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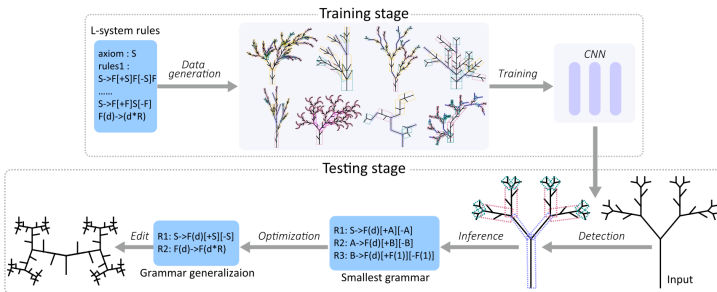


Xu, Xianghao et al. (June 2021). “Inferring CAD Modeling Sequences Using Zone Graphs”. In: *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, pp. 6062–6070. DOI: 10.48550/arXiv.2104.03900. arXiv: 2104.03900.

Other lines of research on inference

► Inferring the generation of an object:

- Inverse procedural modeling: retrieving parameters.¹
- L-systems: retrieving formal rules.² Illustration from (Guo et al. 2020).
- Constructive solid geometry: retrieving sequences of operations.³



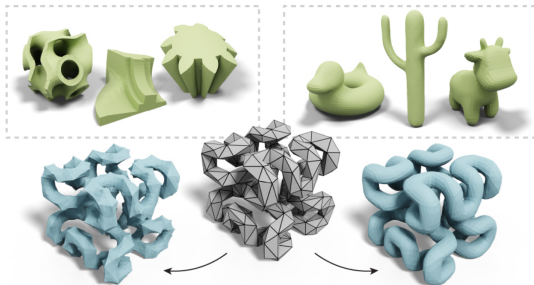
¹Wu et al. 2014; Emilien et al. 2015.

²Santos et al. 2009; Št'ava et al. 2010.

³Sharma et al. 2018; Kania et al. 2020; Xu et al. 2021.

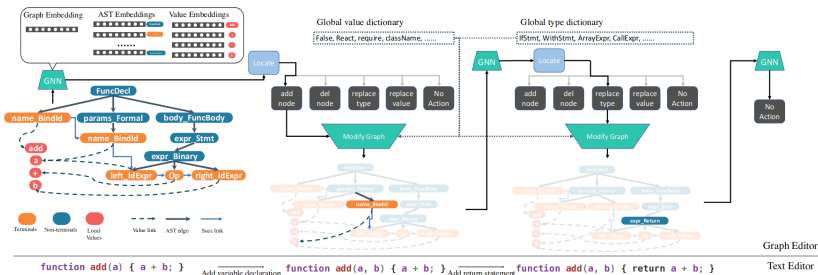
Other lines of research on inference

- ▶ Inferring the generation of an object
- ▶ Pure geometry
 - Retrieve non-linear weights of a Loop-based subdivision scheme for mesh refinement. Illustration from (Liu et al. 2020).



Other lines of research on inference

- ▶ Inferring the generation of an object
- ▶ Pure geometry
- ▶ Graph transformations
 - Domain-based inference mechanism retrieving or exploiting graph transformations.¹ Illustration from (Dinella et al. 2020).



¹Alshanqiti et al. 2016; López-Fernández et al. 2019.

Pseudo-code for TFA

Algorithm 1: Topological folding algorithm

Input: A Gmap G , an orbit type $\langle o \rangle$, and a dart a of G .

Output: The scheme graph $\mathcal{S}$ such that the instantiation $\iota^{\langle o \rangle}(\mathcal{S}, G\langle o \rangle(a))$ maps (h, a) onto a .

```

1  $Q \leftarrow$  empty queue
2  $\mathcal{S} \leftarrow \emptyset$  // Empty scheme graph
3  $h \leftarrow \text{createHook}(\mathcal{S}, \langle o \rangle)$  // Construction of the hook (Step 1)
4  $\text{enqueue}(Q, h)$ 
5 while  $Q$  not empty do // Traversal (Step 2)
6    $m \leftarrow \text{dequeue}(Q)$ 
7   foreach  $d \in (0..n) \setminus \text{orbType}(m)$  do
8      $v \leftarrow \text{arcExt}(G, m, d)$  // Extension of the explicit arcs incident to  $v$  (Step 2.1)
9      $\text{buildOrbType}(G, v)$  // Construction of the orbit type decorating  $v$  (Step 2.2)
10     $\text{enqueue}(Q, v)$ 
11 return  $\mathcal{S}$ 

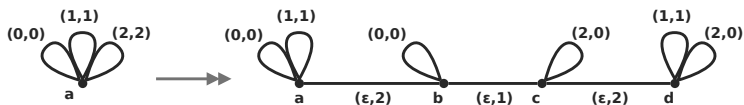
```

Extended rules schemes with support to maps

Cone operation.



► G-map rule scheme

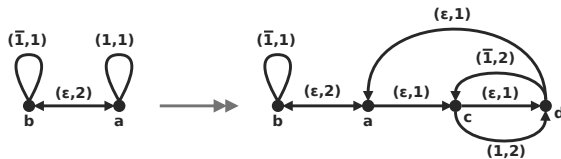


Extended rules schemes with support to maps

Cone operation.



► O-map rule scheme

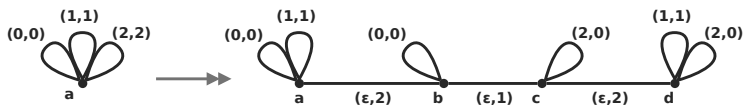


Extended rules schemes with support to maps

Edge rounding operation.



► G-map rule scheme

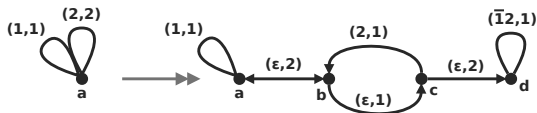


Extended rules schemes with support to maps

Edge rounding operation.

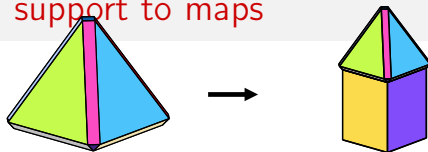


► O-map rule scheme

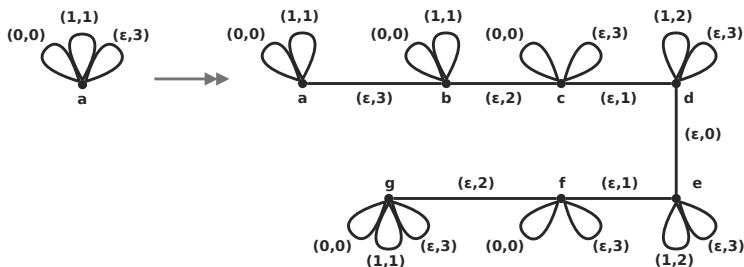


Extended rules schemes with support to maps

Face extrusion operation.

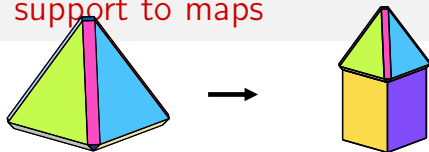


► G-map rule scheme



Extended rules schemes with support to maps

Face extrusion operation.



► O-map rule scheme

