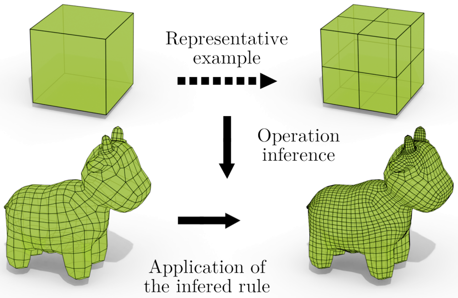
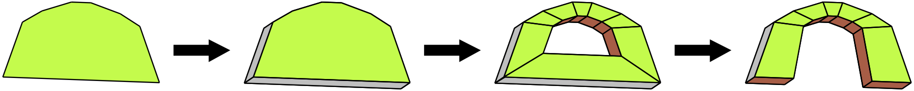
Graph transformation: a tool for designing geometric modeling operations

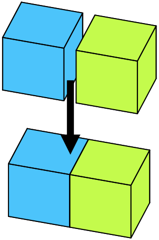


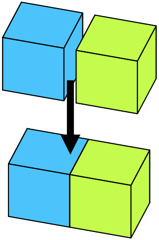
Romain Pascual

joint work with Pascale Le Gall,
Hakim Belhaouari, and
Agnès Arnould

April 11, 2023



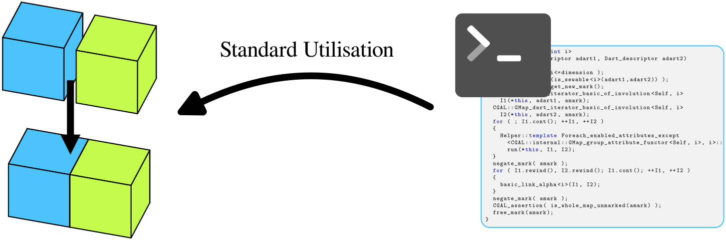




CGAL's sew operation

```
template<unsigned int i>
void sew(Dart_descriptor adart1, Dart_descriptor adart2)

  CGAL_assertion( i<=dimension );
  CGAL_assertion( (is_sewable<i>(adart1,adart2)) );
  size_type amark=get_new_mark();
  CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
    I1(*this, adart1, amark);
  CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
    I2(*this, adart2, amark);
  for ( ; I1.cont(); ++I1, ++I2 )
  {
    Helper::template Foreach_enabled_attributes_except
      <CGAL::internal::GMap_group_attribute_functor<Self, i>, i>::
      run(*this, I1, I2);
  }
  negate_mark( amark );
  for ( I1.rewind(), I2.rewind(); I1.cont(); ++I1, ++I2 )
  {
    basic_link_alpha<i>(I1, I2);
  }
  negate_mark( amark );
  CGAL_assertion( is_whole_map_unmarked(amark) );
  free_mark(amark);
}
```


Standard Utilisation
int i>
riptor adart1, Dart_descriptor adart2)
i<=dimension);
(is_sewable<i>(adart1,adart2)));
get_new_mark();
iterator_basic_of_involution<Self, i>
I1(*this, adart1, amark);
CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
I2(*this, adart2, amark);
for (; I1.cont(); ++I1, ++I2)
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run(*this, I1, I2);
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for (I1.rewind(), I2.rewind(); I1.cont(); ++I1, ++I2)
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basic_link_alpha<i>(I1, I2);
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negate_mark(amark);
CGAL_assertion(is_whole_map_unmarked(amark));
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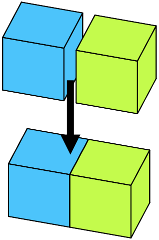
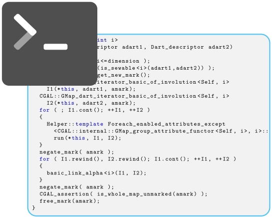
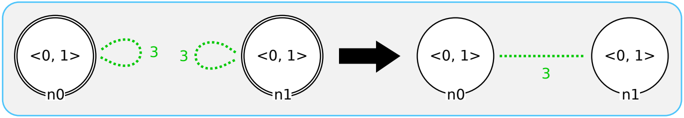
}

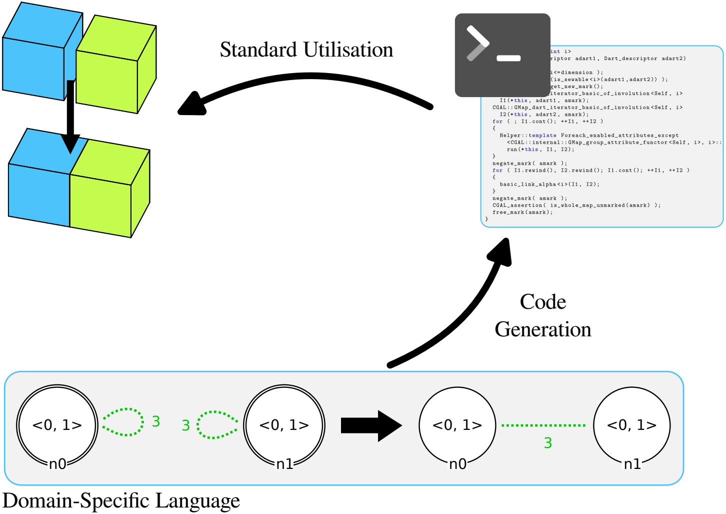


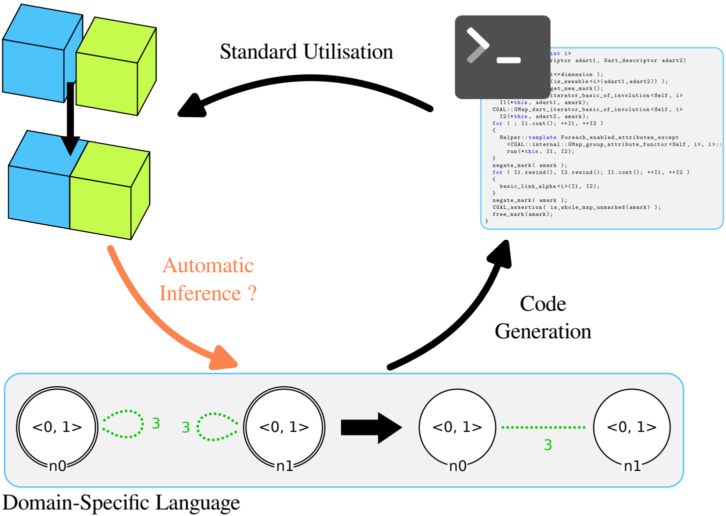
Standard Utilisation





Domain-Specific Language


Standard Utilisation
Code Generation
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1
Domain-Specific Language


Standard Utilisation
Automatic
Inference ?
Code
Generation
<0, 1>
n0
3
3
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n1
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n0
3
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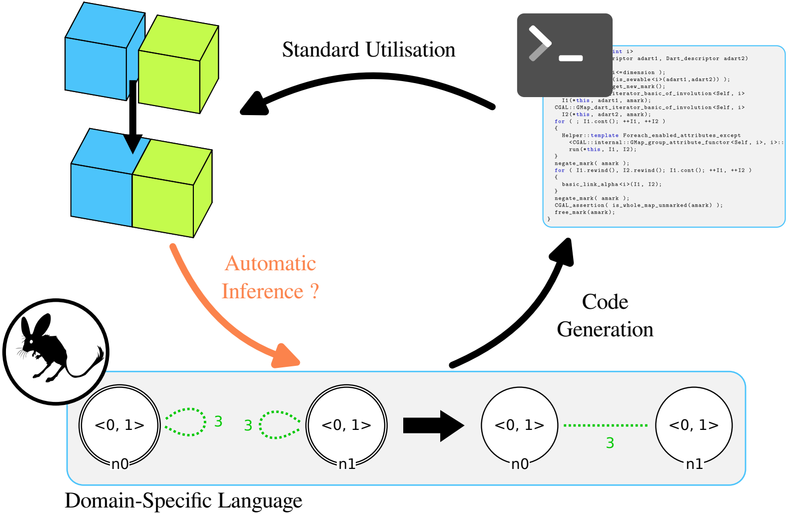
Domain-Specific Language


Standard Utilisation
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<0, 1>
n0
3
3
<0, 1>
n1
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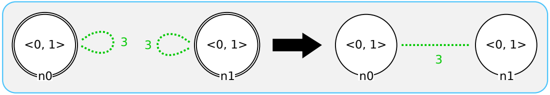
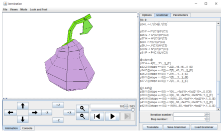
n1
Domain-Specific Language

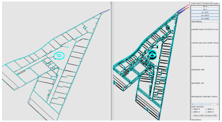
Jerboa's DSL

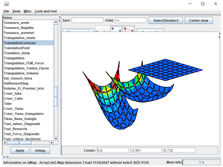
- Main characteristics:
  - Gmaps[1]
  - Syntax analyzer[2]

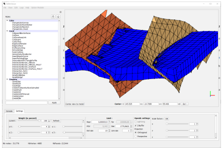


- Successful applications:
  - Plant growth
  - Architecture
  - Spring-mass
  - Geology



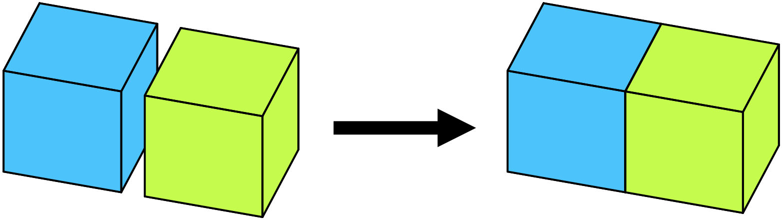


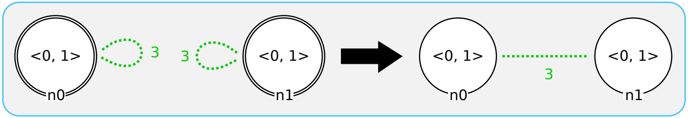




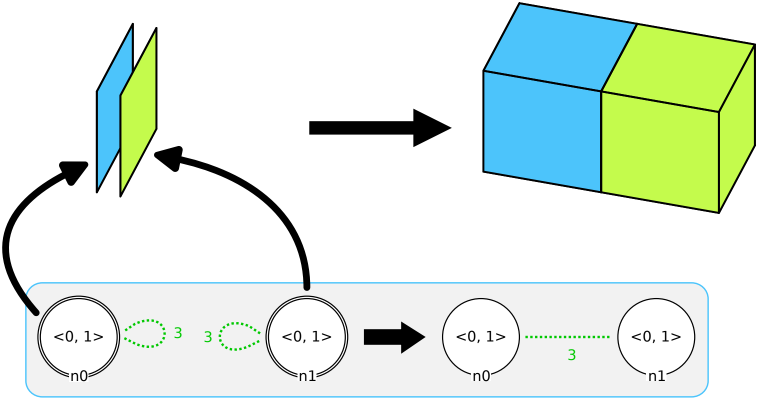
[1] Poudret et al. 2008
[2] Belhaouari et al. 2014

A sneak peek at Jerboa's language

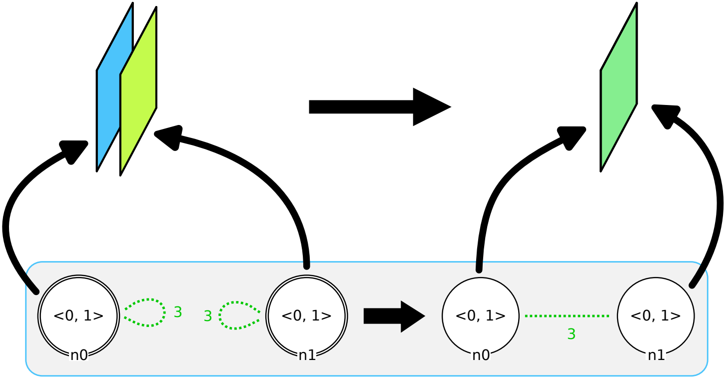




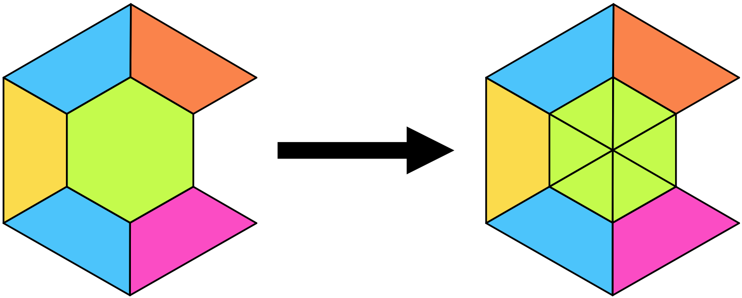
A sneak peek at Jerboa's language

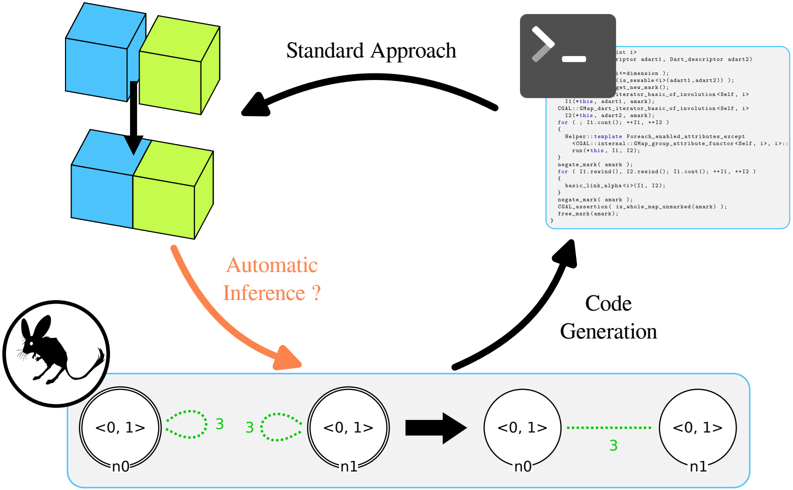


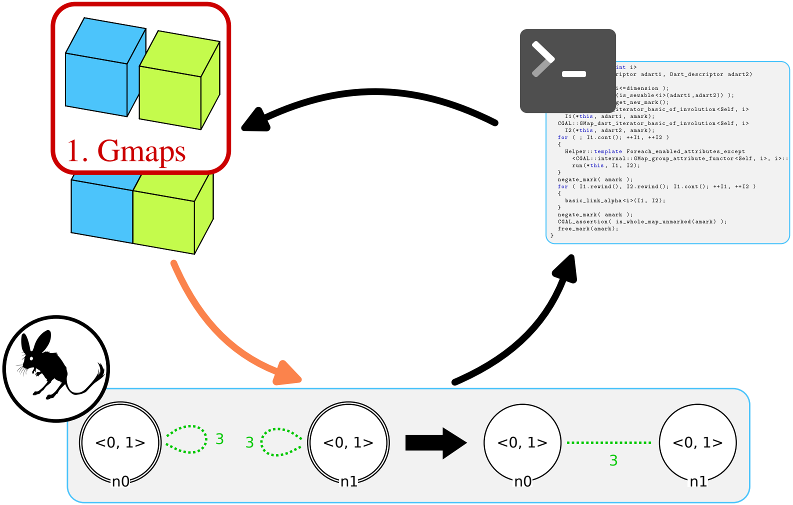
A sneak peek at Jerboa's language



Running example: face triangulation




Standard Approach
Automatic
Inference ?
Code
Generation
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1


1. Gmaps
int i>
riptor adart1, Dart_descriptor adart2)
i<=dimension);
(is_sewable<i>(adart1,adart2)));
get_new_mark();
iterator_basic_of_involution<Self, i>
I1(*this, adart1, amark);
CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
I2(*this, adart2, amark);
for (; I1.cont(); ++I1, ++I2)
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Helper::template Foreach_enabled_attributes_except
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basic_link_alpha<i>(I1, I2);
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CGAL_assertion(is_whole_map_unmarked(amark));
free_mark(amark);
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n0
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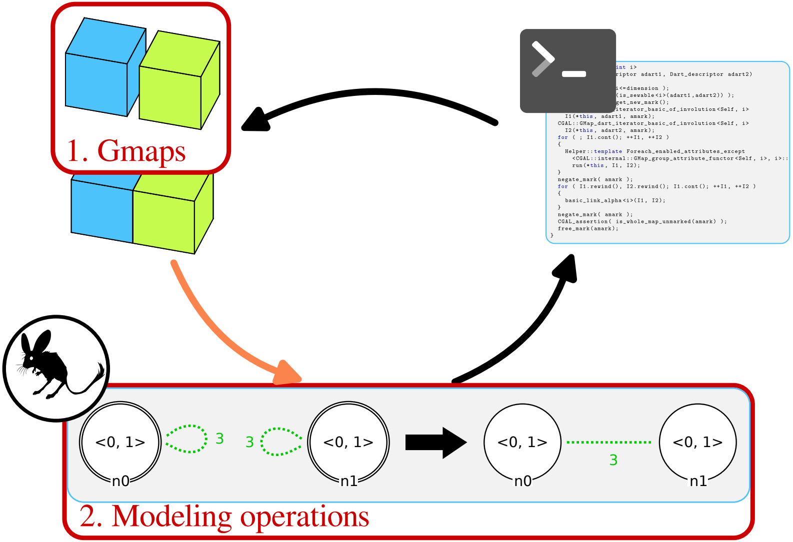
<0, 1>
n0
3
<0, 1>
n1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<0, 1>
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<0, 1>
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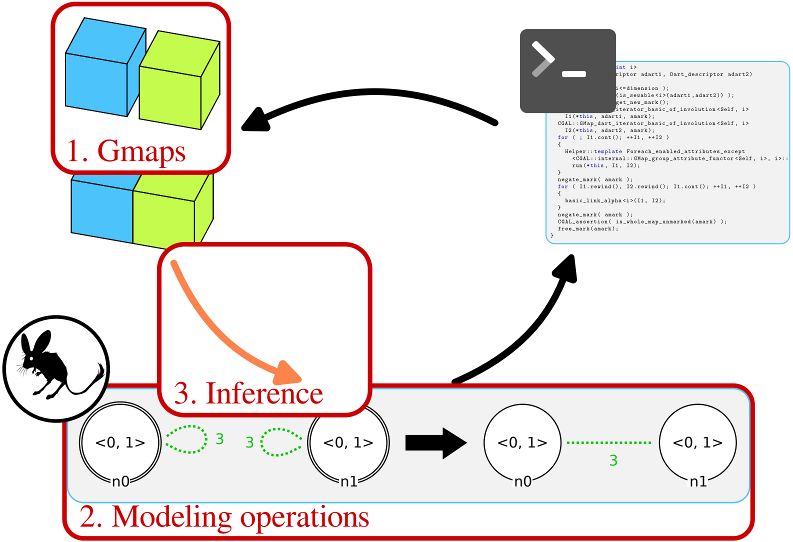
<0, 1>
n1
2. Modeling operations


1. Gmaps
3. Inference
2. Modeling operations
<0, 1>
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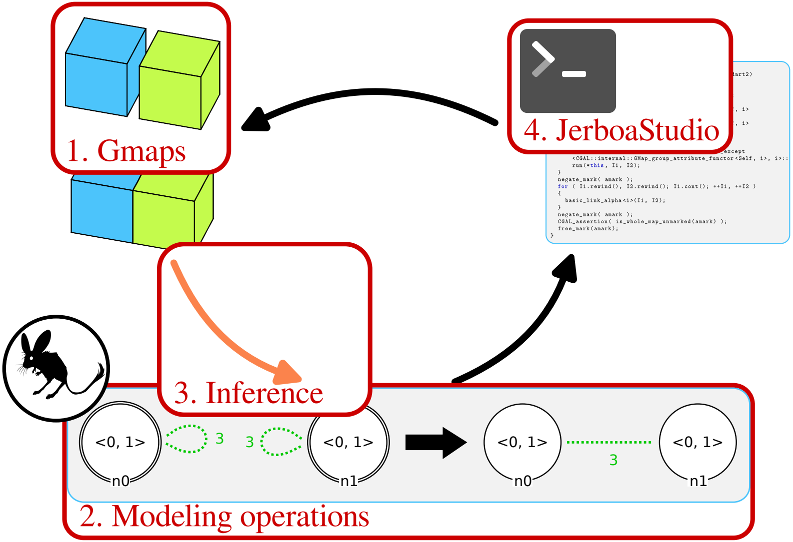

1. Gmaps
4. JerboaStudio
3. Inference
2. Modeling operations
<0, 1>
n0
n1
3
run(*this, I1, I2);
negate_mark(amark);
for (I1.rewind(), I2.rewind(); I1.cont(); ++I1, ++I2)
basic_link_alpha<i>(I1, I2);
CGAL_assertion(is_whole_map_unmarked(amark));
free_mark(amark);

Generalized maps

- Geometric objects are represented with embedded generalized maps.

The category of graphs

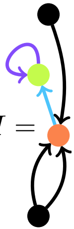
$G =$ 

A graph $G = (V, E, s, t)$:

- a set of nodes V,
- a set of arcs E,
- a source function $s : E \to V$,
- a target arrow $t : E \to V$,

The category of graphs

$G =$ 

$G =$  $\longrightarrow$ $H =$ 

A graph $G = (V, E, s, t)$:

- a set of nodes V,
- a set of arcs E,
- a source function $s : E \to V$,
- a target arrow $t : E \to V$,

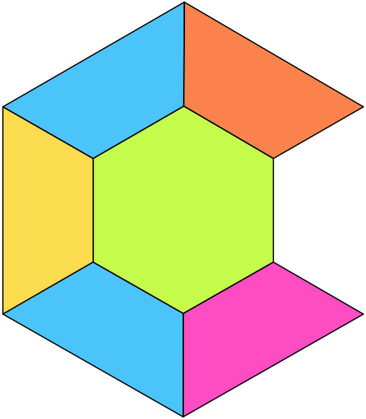
A morphism $G \to H$:

- a node function $V_G \to V_H$,
- an arc function $E_G \to E_H$,

preserving structure.

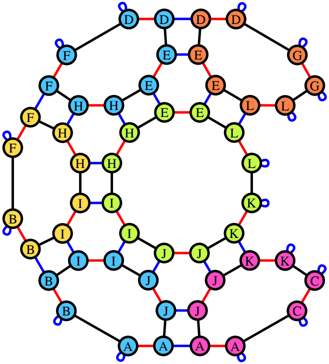
- Decorated with labels, types, and attributes.

Generalized maps[1]



[1] Damiand et al. 2014.

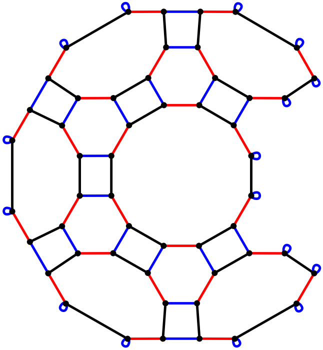
Generalized maps[1]



Color legend: 0, 1, 2.

[1] Damiand et al. 2014.

Generalized maps[1]

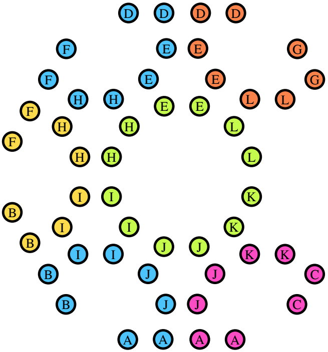


Color legend: 0, 1, 2.

- Topology: graph structure

[1] Damiand et al. 2014.

Generalized maps[1]

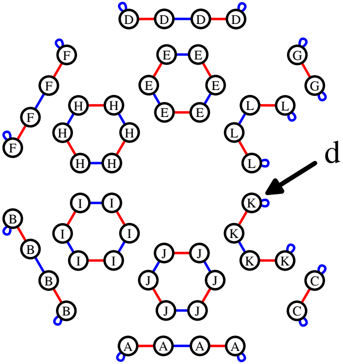


- Topology: graph structure
- Geometry: node attributes

Color legend: 0, 1, 2.

[1]Damiand et al. 2014.

Orbits and topological cells

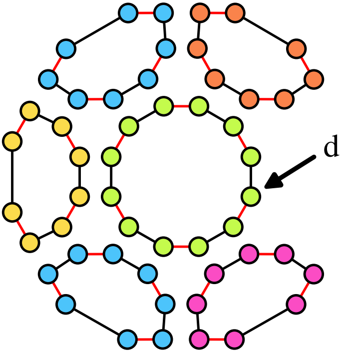


Color legend: 0, 1, 2.

Orbit (encode topological cell):
Graph induced by a subset $\langle o \rangle \subseteq [\![0, n]\!]$ of dimensions.

- positions on vertices (orbits $\langle 1, 2 \rangle$).

Orbits and topological cells



Orbit (encode topological cell):
Graph induced by a subset $\langle o \rangle \subseteq [\![0, n]\!]$ of dimensions.

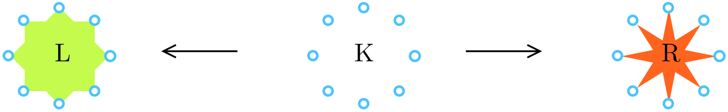
- positions on vertices (orbits $\langle 1, 2 \rangle$).
- colors on faces (orbits $\langle 0, 1 \rangle$).

Color legend: 0, 1, 2.

Formalizing modeling operations

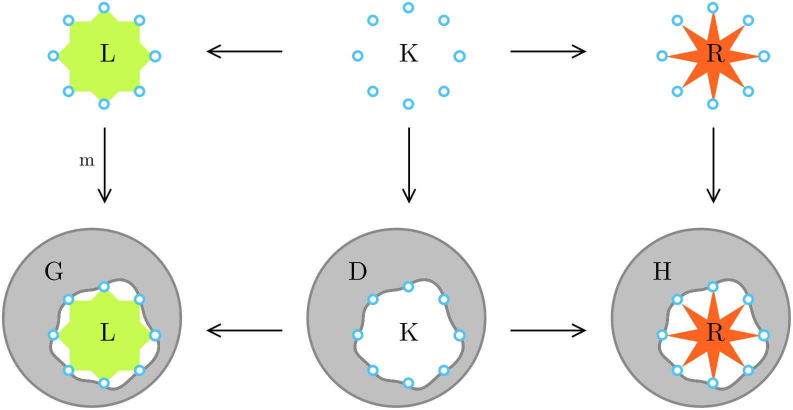
- Operations on Gmaps are designed as graph rewriting rules.

Graph transformation rules[1]



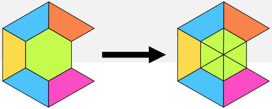
[1] Rozenberg 1997; Ehrig et al. 2006; Heckel et al. 2020.

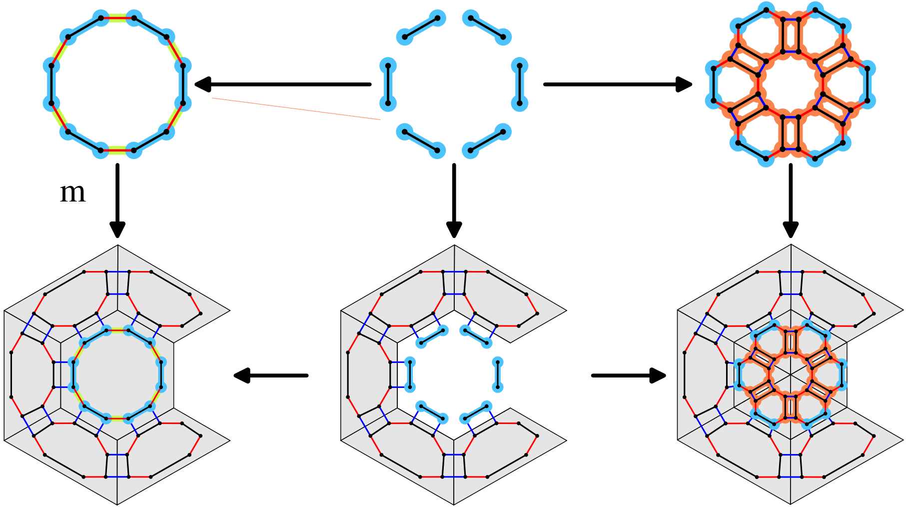
Graph transformation rules[1]



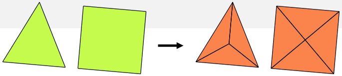
[1] Rozenberg 1997; Ehrig et al. 2006; Heckel et al. 2020.

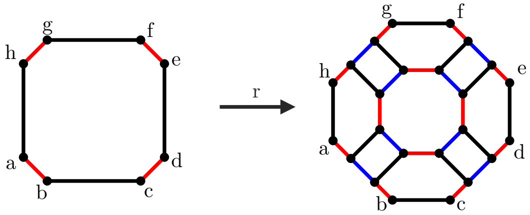
Rewriting Gmaps



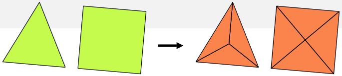


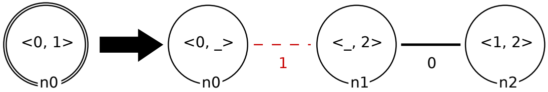
Orbit rewriting

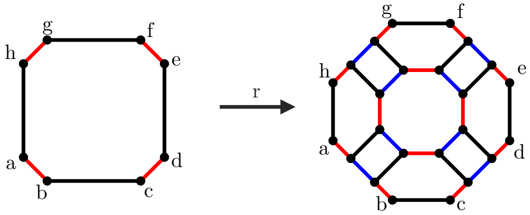




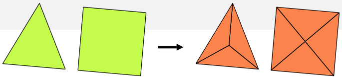
Orbit rewriting

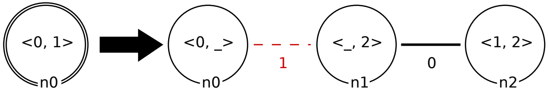


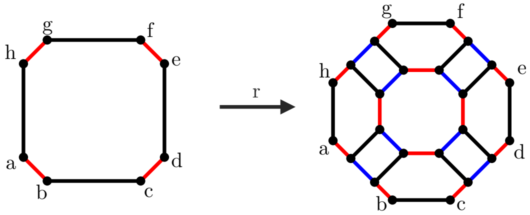




Orbit rewriting







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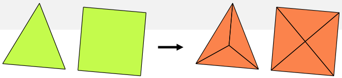
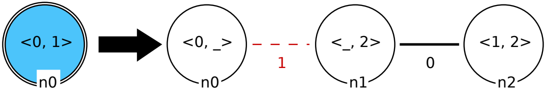
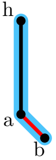
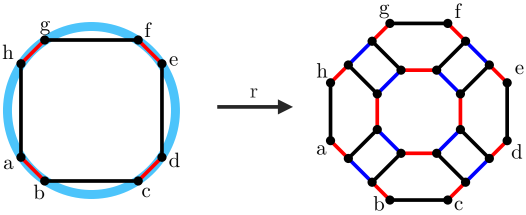
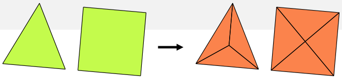


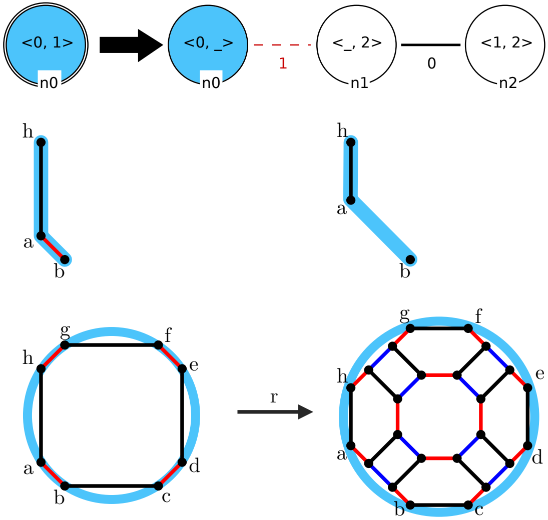




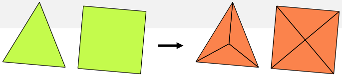


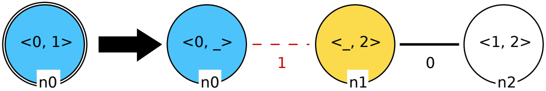
Orbit rewriting

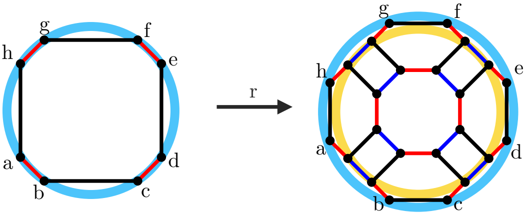




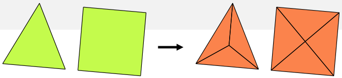
Orbit rewriting

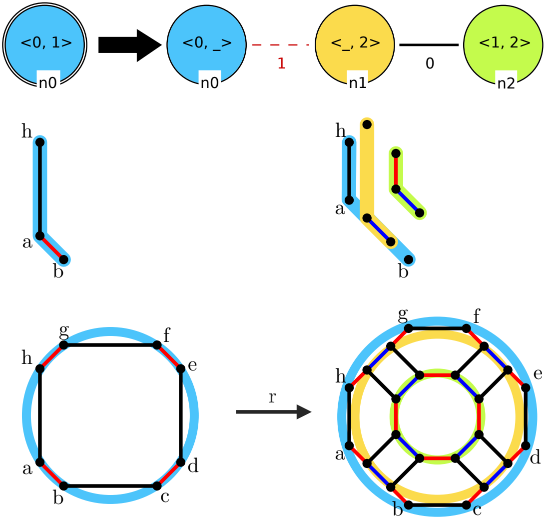




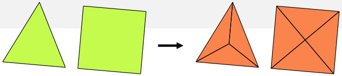


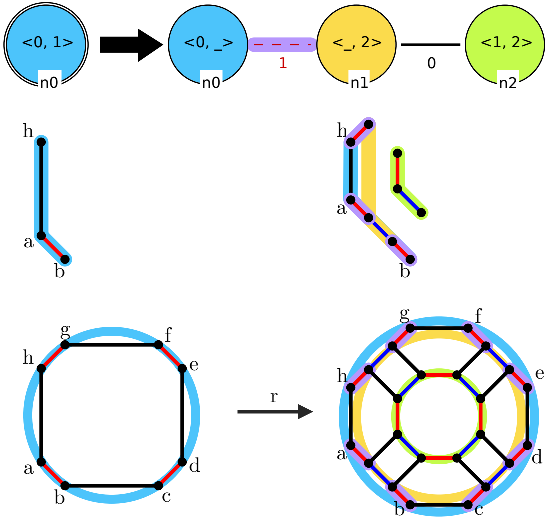
Orbit rewriting





Orbit rewriting





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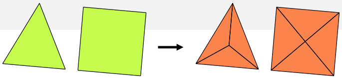
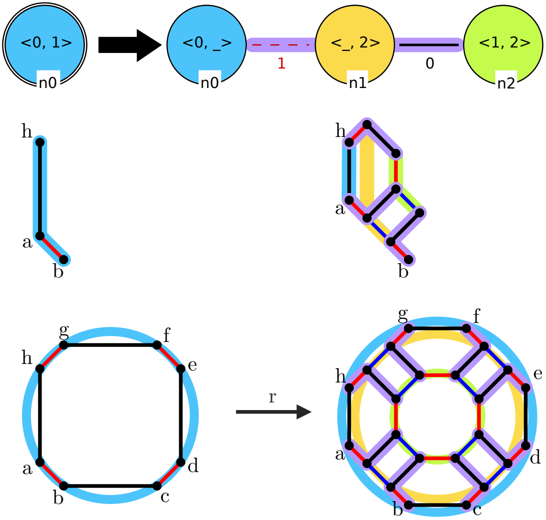
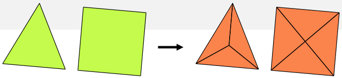
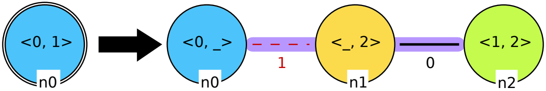
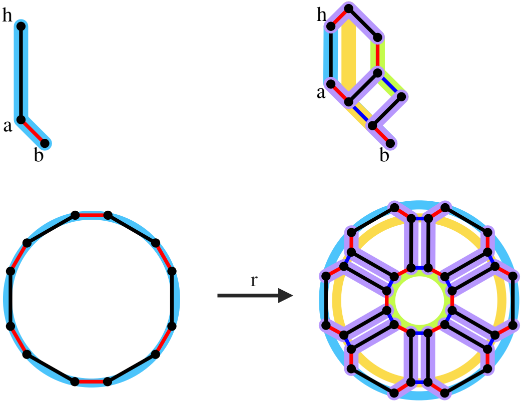




Orbit rewriting







The complete construction

$$
\begin{tikzcd}
1_{\mathbb{W}\times\mathbb{D}} & & & \\
E_{\mathbb{D}}(P_v) \arrow[u, "!_{E_{\mathbb{D}}(P_v)}"] & L \arrow[ul, "!_L"'] & K \arrow[l, hook'] \arrow[r, hook] \arrow[ull, "!_K"'] & R \arrow[ulll, "!_R"'] \\
P_v \arrow[u, Rightarrow, "E_{\mathbb{D}}"] \arrow[d, hook, "p_v"'] & E_{\mathbb{D}}(P_v) \times L \arrow[u] \arrow[ul] & E_{\mathbb{D}}(P_v) \times K \arrow[l, hook'] \arrow[r, hook] \arrow[u] \arrow[ull] & E_{\mathbb{D}}(P_v) \times R \arrow[u] \arrow[ulll] \\
\mathbb{P}(\mathcal{G}) & \iota(L, P_v) \arrow[from=u, Rightarrow, "\pi_{\mathbb{D}}"] \arrow[d, hook, "m"] & \iota(K, P_v) \arrow[from=u, Rightarrow, "\pi_{\mathbb{D}}"] \arrow[l, hook'] \arrow[r, hook] \arrow[d, hook] & \iota(R, P_v) \arrow[from=u, Rightarrow, "\pi_{\mathbb{D}}"] \arrow[d, hook] \\
 & \mathcal{G} \arrow[ul, Rightarrow, "\mathbb{P}"] & \mathcal{D} \arrow[l, hook'] \arrow[r, hook] & \mathcal{H}
\end{tikzcd}
$$

The complete construction

$$
\begin{tikzcd}
1_{\mathbb{W}\times\mathbb{D}} & & & \\
E_{\mathbb{D}}(P_v) \arrow[u, "!_{E_{\mathbb{D}}(P_v)}"] & \boxed{\begin{array}{l} L \hookleftarrow K \hookrightarrow R \\ \text{Input 1: Rule scheme} \end{array}} \arrow[ul, two heads, "!_L"] \arrow[ul, two heads, "!_K"] \arrow[ul, two heads, "!_R"] & & \\
P_v \arrow[u, Rightarrow, "E_{\mathbb{D}}"] \arrow[d, hook, "p_v"'] & E_{\mathbb{D}}(P_v)\times L \arrow[ul, two heads] \arrow[u, dash] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v)\times K \arrow[l, hook'] \arrow[r, hook] \arrow[ull, two heads] \arrow[ul, dash] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v)\times R \arrow[ulll, two heads] \arrow[ull, dash] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] \\
\mathbb{P}(\mathcal{G}) & \iota(L,P_v) \arrow[d, hook, "m"] & \iota(K,P_v) \arrow[l, hook'] \arrow[r, hook] \arrow[d, hook] & \iota(R,P_v) \arrow[d, hook] \\
 & \boxed{\begin{array}{l} \mathcal{G} \\ \text{Input 2: Gmap} \end{array}} \arrow[ul, Rightarrow, two heads, "\mathbb{P}"] & \mathcal{D} \arrow[l, hook'] \arrow[r, hook] & \boxed{\begin{array}{l} \mathcal{H} \\ \text{Output: Gmap} \end{array}}
\end{tikzcd}
$$

The complete construction

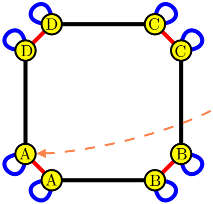
$$
\begin{tikzcd}
1_{\mathbb{W}\times\mathbb{D}} & & & \\
E_{\mathbb{D}}(P_v) \arrow[u, "!_{E_{\mathbb{D}}(P_v)}"] & {\color{red}L} \arrow[ul, "!_L"] & {\color{red}K} \arrow[l, hook'] \arrow[r, hook] \arrow[ull, "!_K"] & {\color{red}R} \arrow[ulll, "!_R"] \\
P_v \arrow[u, Rightarrow, "E_{\mathbb{D}}"] \arrow[d, hook, "p_v"'] & E_{\mathbb{D}}(P_v)\times L \arrow[u] \arrow[ul] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v)\times K \arrow[l, hook'] \arrow[r, hook] \arrow[u] \arrow[ull] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] & E_{\mathbb{D}}(P_v)\times R \arrow[u] \arrow[ulll] \arrow[d, Rightarrow, "\pi_{\mathbb{D}}"] \\
\mathbb{P}(\mathcal{G}) & \iota(L,P_v) \arrow[d, hook, "m"] \arrow[l, Rightarrow, "\mathbb{P}"] & \iota(K,P_v) \arrow[l, hook'] \arrow[r, hook] \arrow[d, hook] & \iota(R,P_v) \arrow[d, hook] \\
 & {\color{red}\mathcal{G}} & \mathcal{D} \arrow[l, hook'] \arrow[r, hook] & {\color{red}\mathcal{H}}
\end{tikzcd}
$$

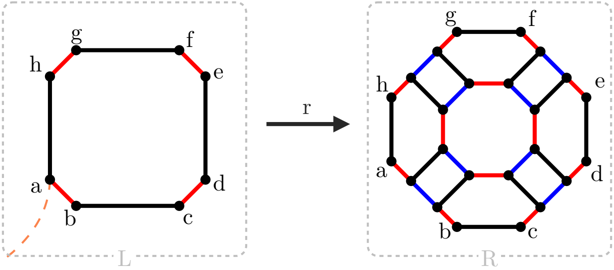
'Standard' DPO step

Modifying geometric values[1]

[1]Bellet et al. 2017.

Modifying geometric values[1]



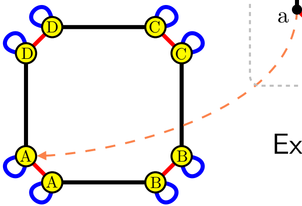


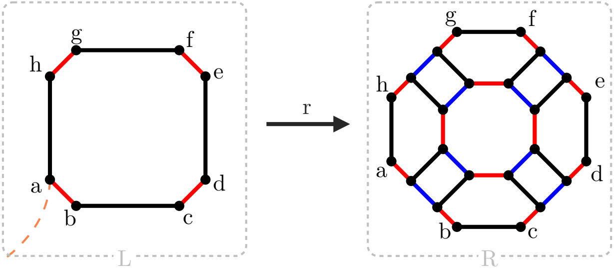
Algebraic data types:

- `point3D`, `colorRGB`, ...
- add, middle, scale, ...

[1] Bellet et al. 2017.

Modifying geometric values[1]



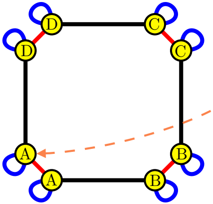


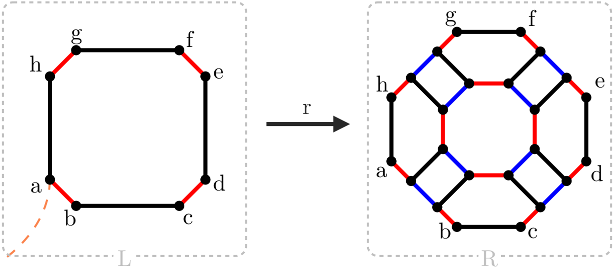
Extended with topological operators:

- Neighbor operator:
  - $a@0@1@0.position = f.position = C$
  - $a@1@0.color = c.color =$ ●

[1] Bellet et al. 2017.

Modifying geometric values[1]



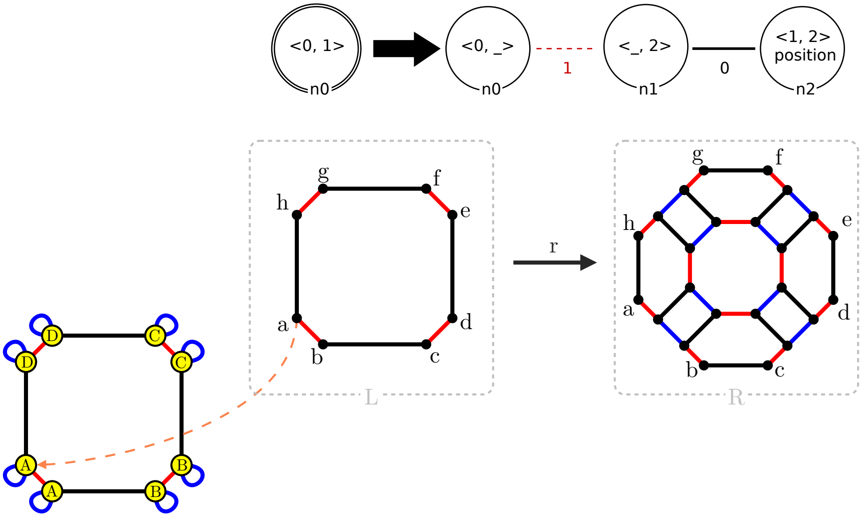


Extended with topological operators:

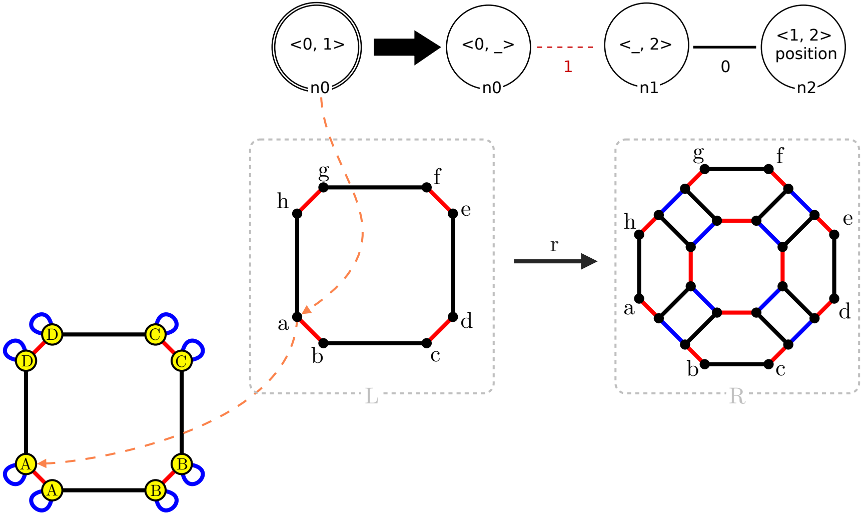
- Neighbor operator:
- Collect operator:
  - ▶ $position_{\langle 0,1 \rangle}(a) = \{A, B, C, D\}$
  - ▶ $color_{\langle 0,1 \rangle}(a) = \{$ ● $\}$

[1]Bellet et al. 2017.

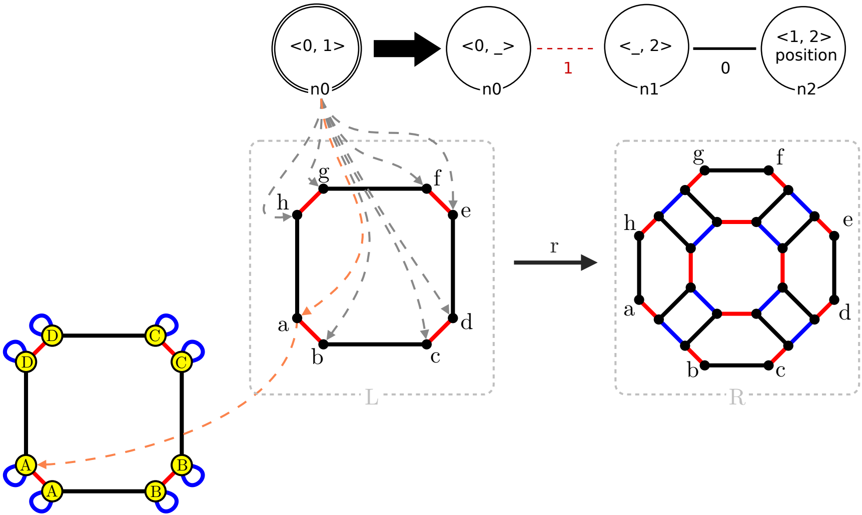
Extension to schemes



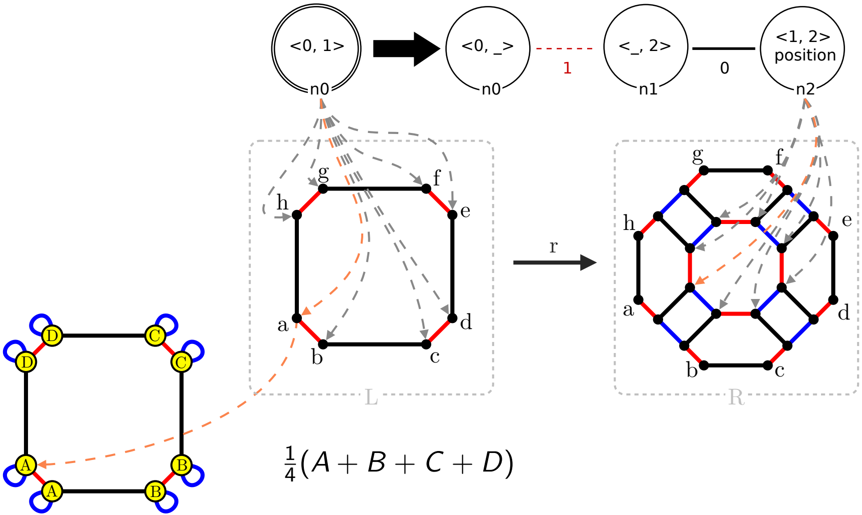
Extension to schemes



Extension to schemes

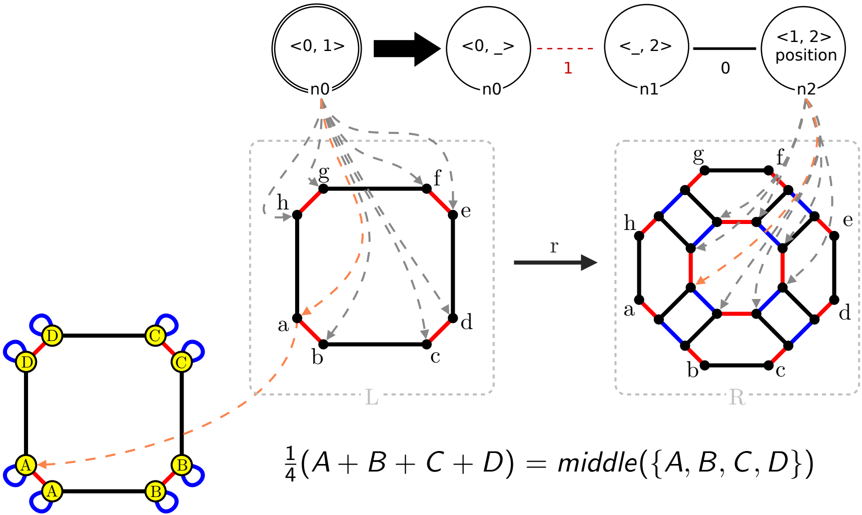


Extension to schemes



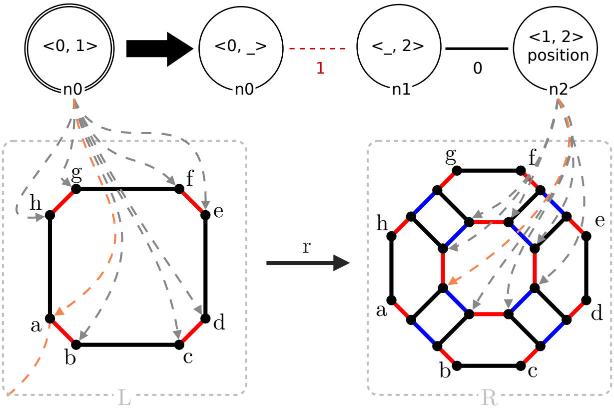
$$\frac{1}{4}(A+B+C+D)$$

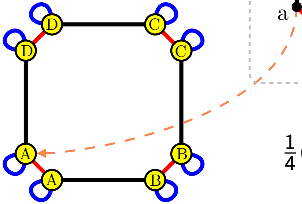
Extension to schemes



$$\frac{1}{4}(A + B + C + D) = middle(\{A, B, C, D\})$$

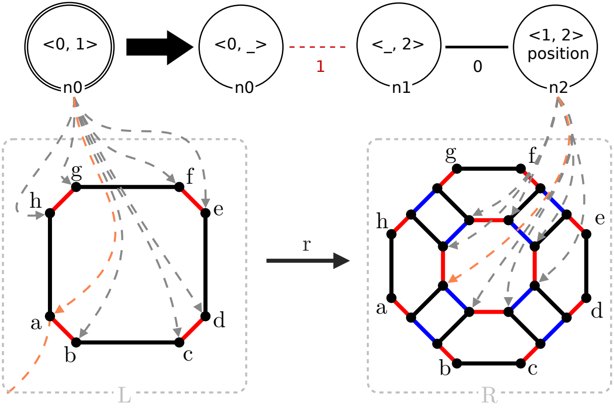
Extension to schemes

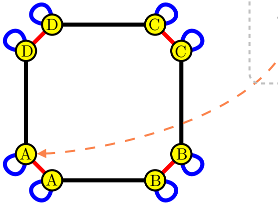




$$\frac{1}{4}(A+B+C+D) = middle(\{A, B, C, D\})$$
$$= middle(position_{\langle 0,1\rangle}(a))$$

Extension to schemes





$$\begin{aligned}\tfrac{1}{4}(A+B+C+D) &= \mathit{middle}(\{A,B,C,D\}) \\ &= \mathit{middle}(\mathit{position}_{\langle 0,\textcolor{red}{1}\rangle}(a)) \\ &= \mathit{middle}(\mathit{position}_{\langle 0,\textcolor{red}{1}\rangle}(n0))\end{aligned}$$

Consistency preservation

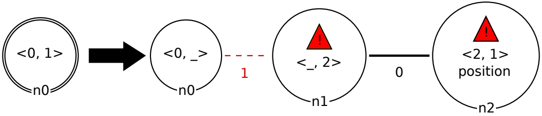
Modifying a well-formed object should produce a well-formed object.

Feedback to the rule designer.

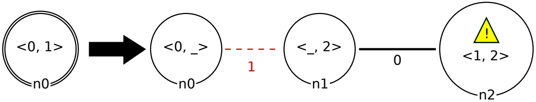
Modifying a well-formed object should produce a well-formed object.

Feedback to the rule designer.

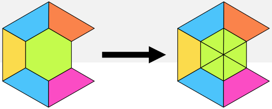
- Topological inconsistencies



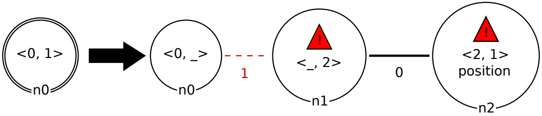
- Geometric inconsistencies



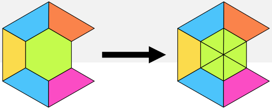
Breaking the topological consistency



Constraint: 0202 paths should be cycles.



Breaking the topological consistency



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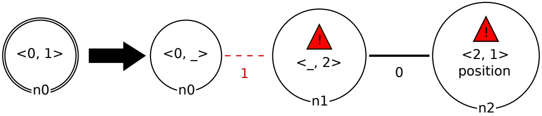
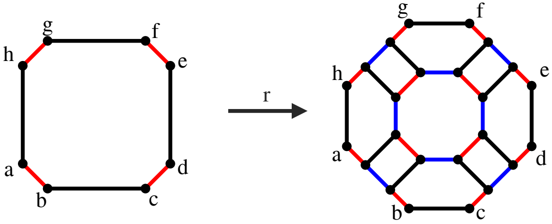
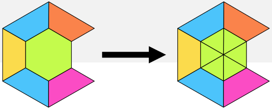




Breaking the topological consistency



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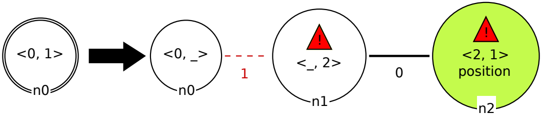
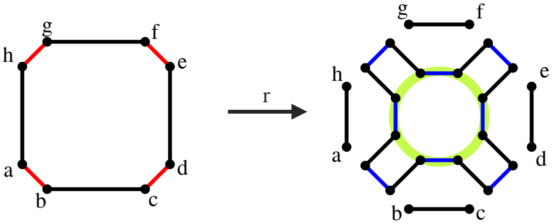
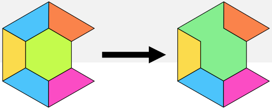
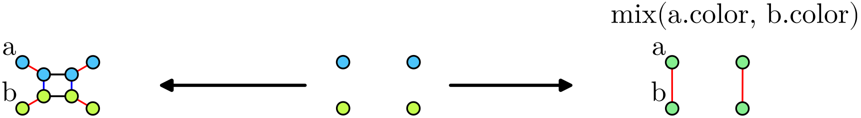




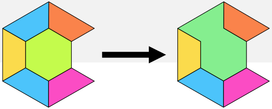
Breaking the geometric consistency

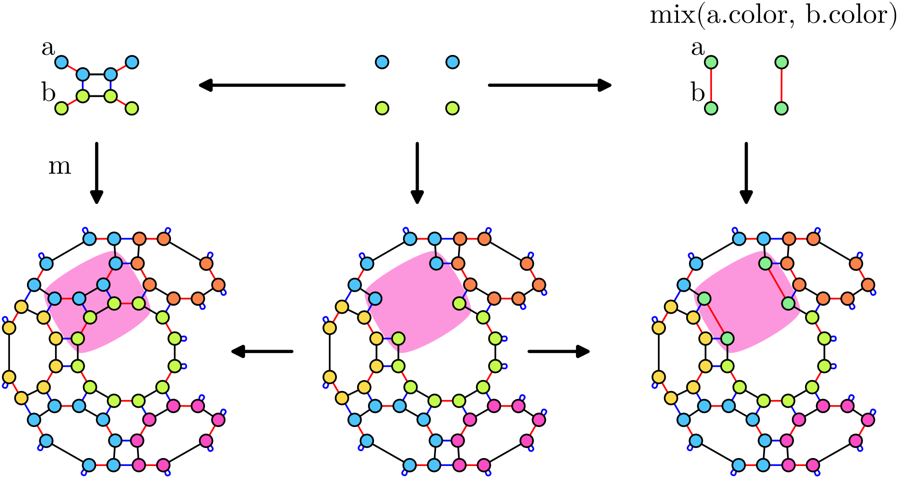


Constraint: nodes in a $\langle 0, 1 \rangle$-orbit should have the same color.

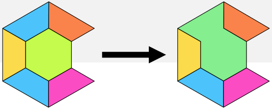


Breaking the geometric consistency





Breaking the geometric consistency



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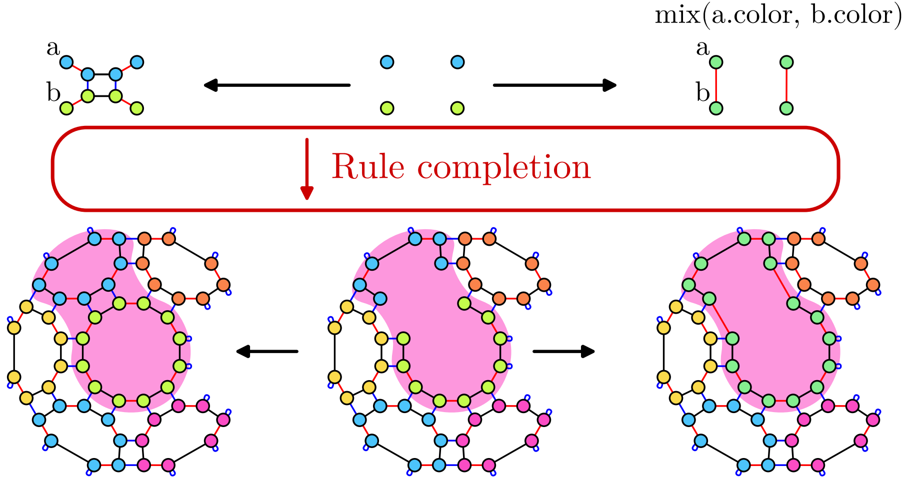
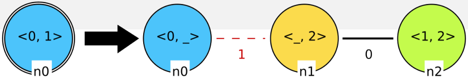
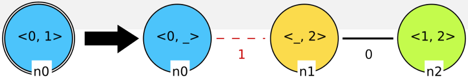


Formalizing Jerboa's DSL



- Jerboa's DSL with categorical constructions: products, attributes, completion.
- Weaker consistency conditions.
- Unified framework to study generalized and oriented maps.

Formalizing Jerboa's DSL



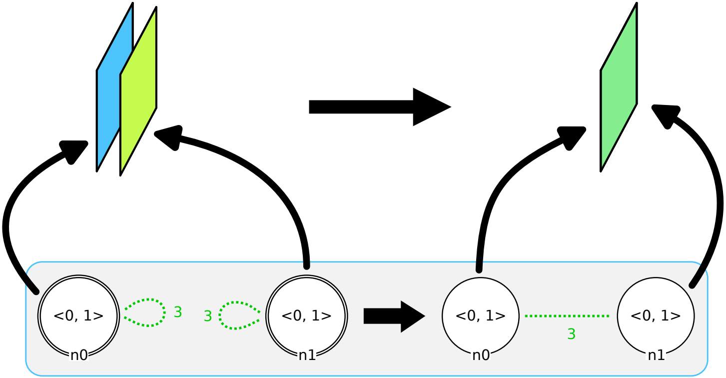
- Jerboa's DSL with categorical constructions: products, attributes, completion.
- Weaker consistency conditions.
- Unified framework to study generalized and oriented maps.

Main lesson:
a DSL allows for the safe design of geometric modeling operations.

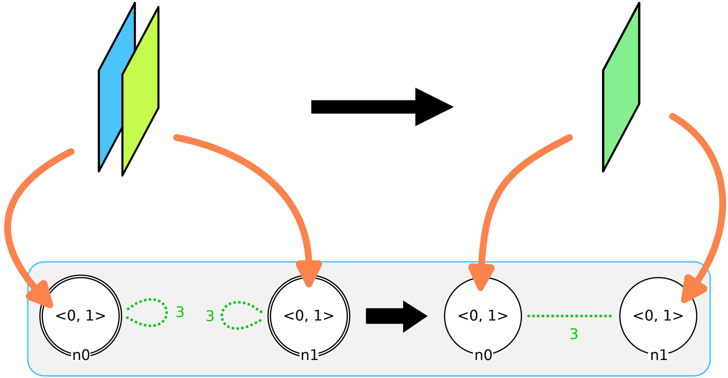
- Romain Pascual et al. (2022b). "Topological consistency preservation with graph transformation schemes". In: Science of Computer Programming. DOI: 10.1016/j.scico.2021.102728
- Agnès Arnould et al. (2022). "Preserving consistency in geometric modeling with graph transformations". In: Mathematical Structures in Computer Science. DOI: 10.1017/S0960129522000226

Inferring geometric modeling operations

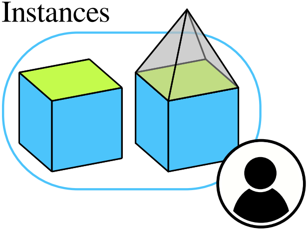
- Retrieving the operation described by an example.

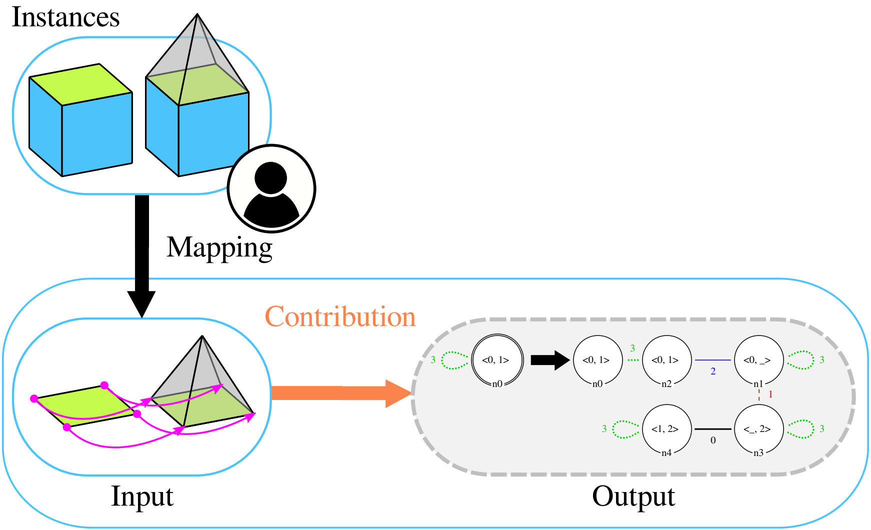


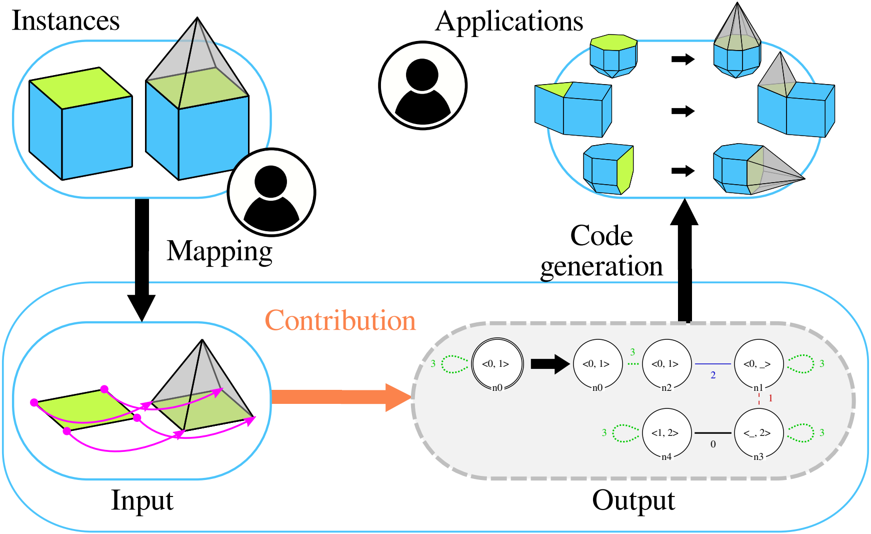
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1


<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1

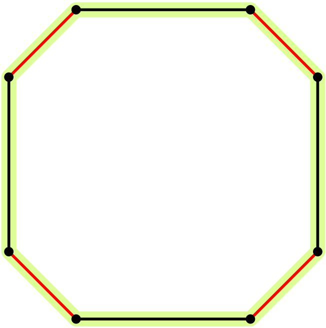
Instances




Instances
Mapping
Contribution
<0, 1>
n0
<0, 1>
n0
3
<0, 1>
n2
2
<0, _>
n1
1
<1, 2>
n4
0
<_, 2>
n3
Input
Output


Instances
Applications
Mapping
Code
generation
Contribution
<0, 1>
n0
<0, 1>
n0
3
<0, 1>
n2
2
<0, _>
n1
3
3
1
3
<1, 2>
n4
0
<_, 2>
n3
3
Input
Output

Folding a joint representation of the rule



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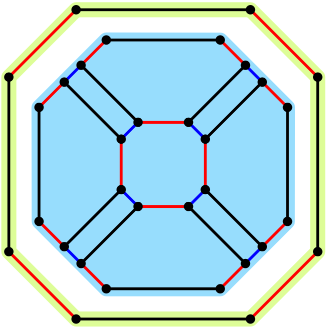

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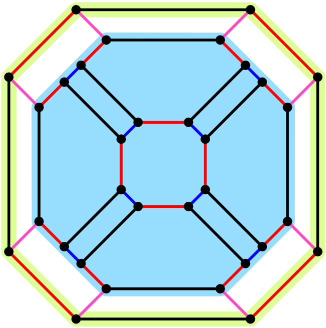
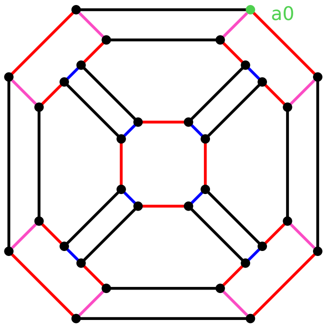


Color legend: 0, 1, 2, κ.

Folding a joint representation of the rule



Color legend: 0, 1, 2, κ.

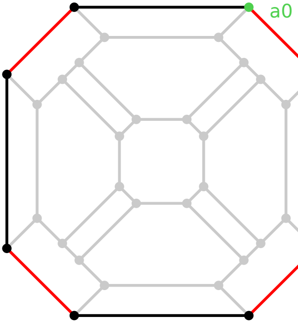
Topological folding algorithm: graph traversal folding nodes and arcs.

Input:

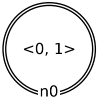
- two partial Gmaps
- preservation links
- a dart
- an orbit type.

▶ Orbit type $\langle 0, 1 \rangle$ and dart $a0$.

Execution

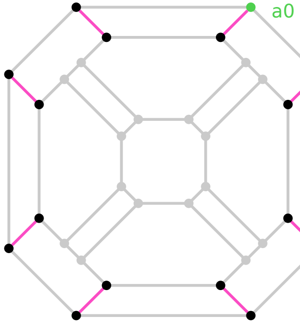


Hook (orbit $\langle 0, 1 \rangle$).

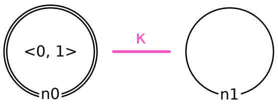


Color legend: 0, 1, 2, κ.

Execution



Folding the arcs.



Color legend: 0, 1, 2, κ.

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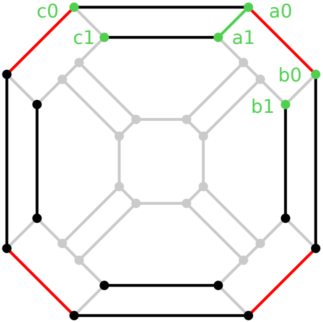
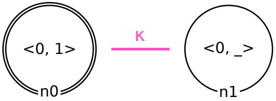
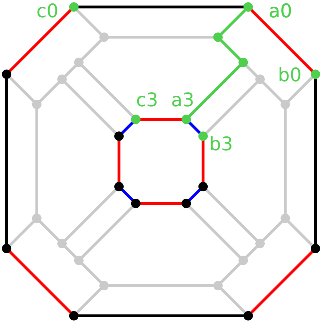


Folding a node.



Color legend: 0, 1, 2, κ.

Execution



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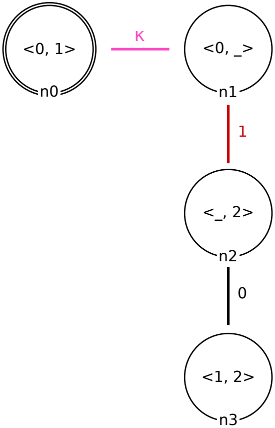
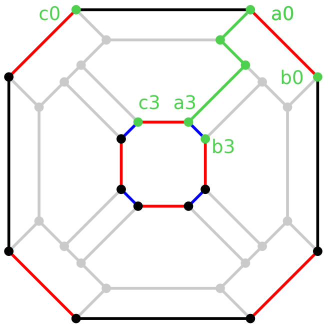
The algorithm terminates.

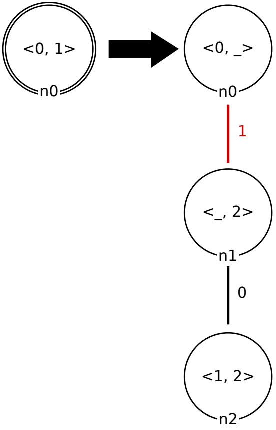


Execution



Color legend: 0, 1, 2, κ.

Splitting the joint representation.

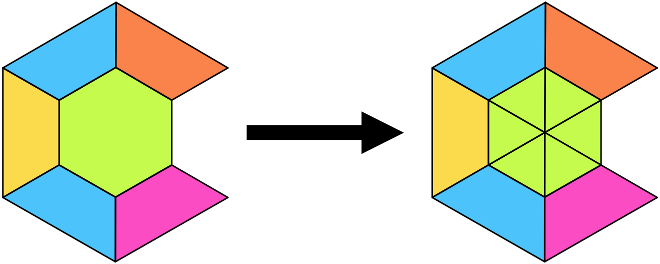


Results

Correctness: The algorithm returns a topological folding of the rule if it exists and halts otherwise.

- Romain Pascual et al. (2022a). "Inferring topological operations on generalized maps: Application to subdivision schemes". In: Graphics and Visual Computing. DOI: 10.1016/j.gvc.2022.200049

▶ What about cases where we cannot fold the rule? Orbit $\langle 0, 1, 2 \rangle$.



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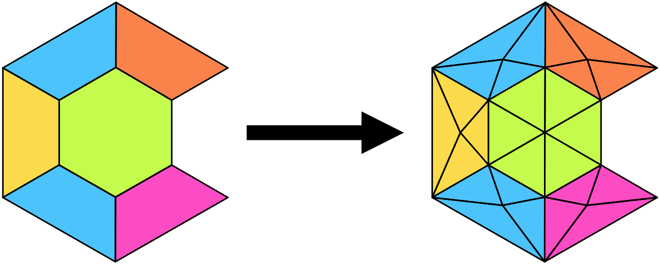

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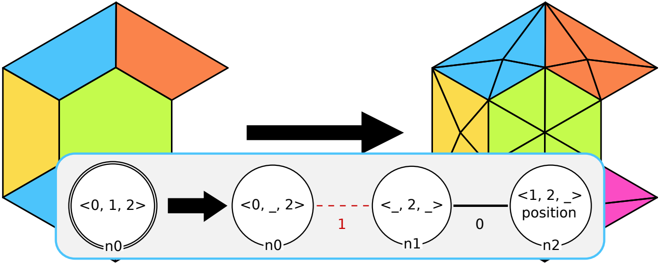

Method (inference of positions)

Hypothesis: Affine combinations of positions.

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For each vertex in C, we want a position p expressed as:

$$p = \sum_{i=0}^{k} w_i p_i + t$$

where:

- p : target position (known)

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- w_i: weight (unknown)

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where:

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- p_i: position of the initial vertex i (known)
- w_i: weight (unknown)
- t : translation (unknown)

Need for abstraction on schemes

$(w_i)_{0 \leq i \leq k}$ such that: $p = \sum_{i=0}^{k} w_i p_i + t$

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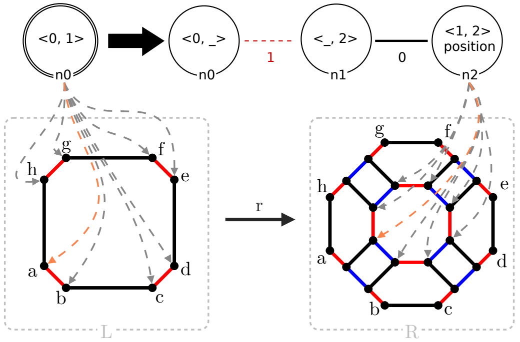

Rule schemes abstract topological cells.

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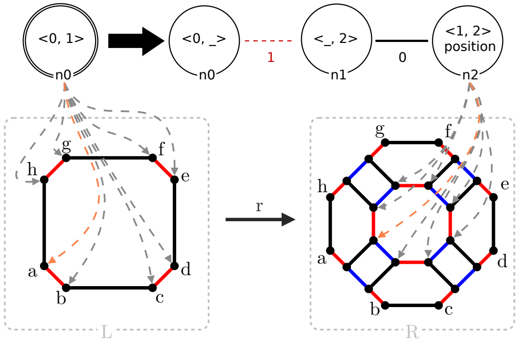
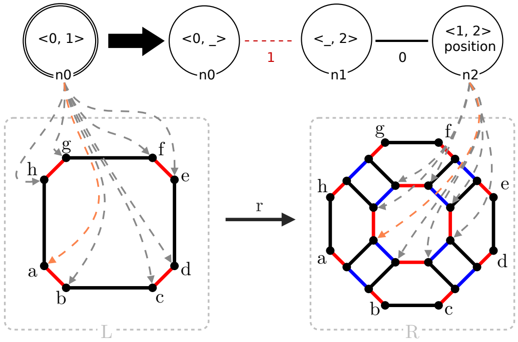


Rule schemes abstract topological cells.

Issue: Darts share the same expression.

$(w_i)_{0 \leq i \leq k}$ such that: $p = \sum_{i=0}^{k} w_i p_i + t$



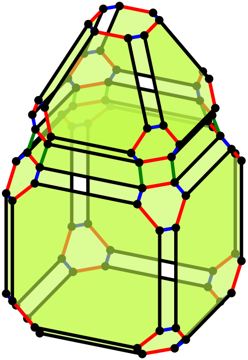
Rule schemes abstract topological cells.

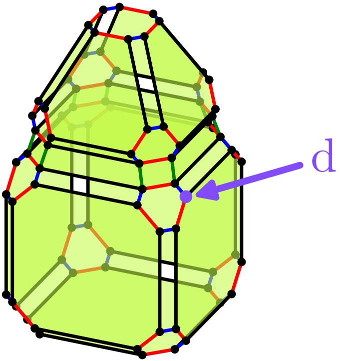
Issue: Darts share the same expression.

Solution: Exploit the topology.

- Use points of interest.

Points of interest

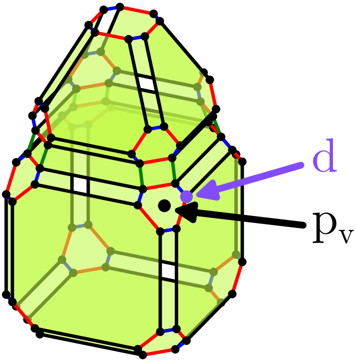



d

Points of interest

with

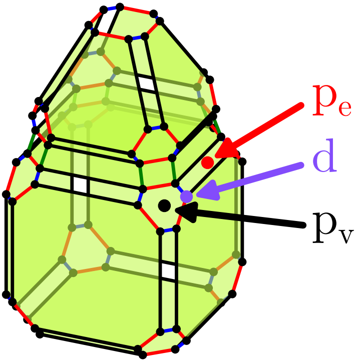
- p_v : vertex



$$p_v = middle(position_{\langle 1,2,3\rangle}(d))$$

with

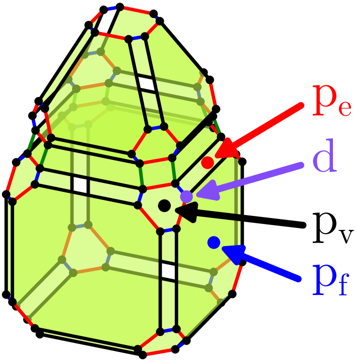
- p_v : vertex
- p_e : edge midpoint



$$p_e = middle(position_{\langle 0,2,3 \rangle}(d))$$

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter

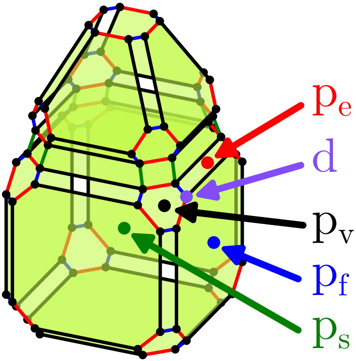


$$p_f = middle(position_{\langle 0,1,3\rangle}(d))$$

Points of interest

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter
- p_s : volume barycenter

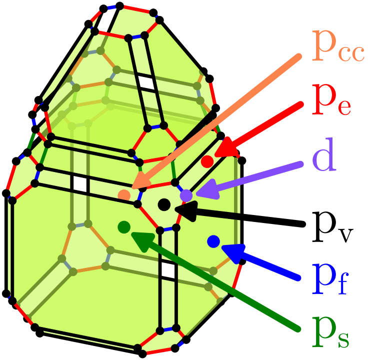


$$p_s = middle(position_{\langle 0,1,2\rangle}(d))$$

Points of interest

with

- p_v : vertex
- p_e : edge midpoint
- p_f : face barycenter
- p_s : volume barycenter
- p_{cc} : CC barycenter



$$p_{cc} = middle(position_{\langle 0,1,2,3\rangle}(d))$$

Points of interest

with

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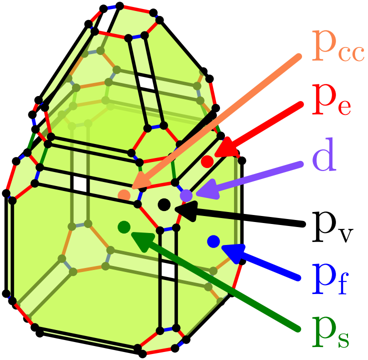
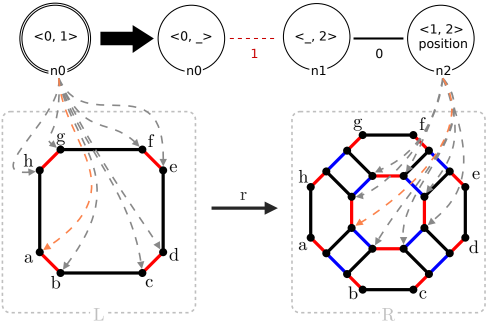
- p_e : edge midpoint
- p_f : face barycenter
- p_s : volume barycenter
- p_{cc} : CC barycenter



The system becomes:

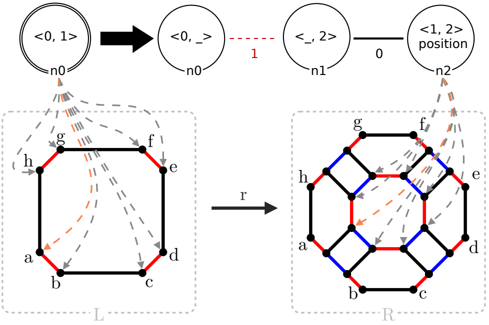
$$p = w_v p_v + w_e p_e + w_f p_f + w_s p_s + w_{cc} p_{cc} + t$$

Illustration



Position of *n*2 as a function of *n*0.

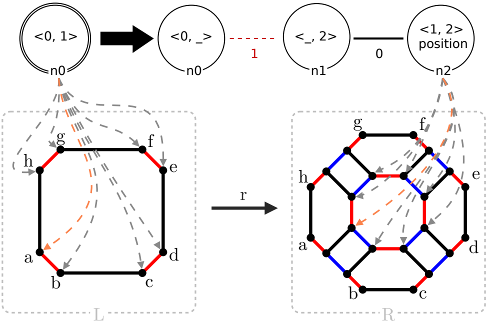
Illustration



Position of $n2$ as a function of $n0$.

$$n2.position = \underbrace{w_v\, n0.p_v}_{vertex} + \underbrace{w_e\, n0.p_e}_{edge} + \underbrace{w_f\, n0.p_f}_{face} + \underbrace{w_s\, n0.p_s}_{volume} + \underbrace{w_{cc}\, n0.p_{cc}}_{cc} + t$$

Illustration



Position of $n2$ as a function of $n0$.

- One equation per dart (8 darts).

$$n2.position = \underbrace{w_v\, n0.p_v}_{vertex} + \underbrace{w_e\, n0.p_e}_{edge} + \underbrace{w_f\, n0.p_f}_{face} + \underbrace{w_s\, n0.p_s}_{volume} + \underbrace{w_{cc}\, n0.p_{cc}}_{cc} + t$$

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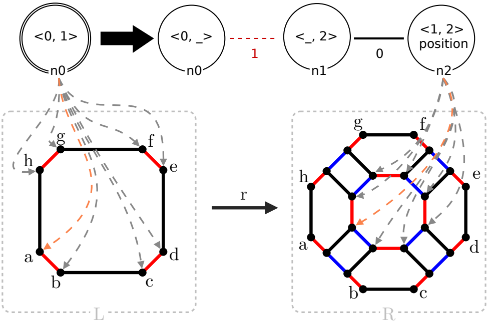
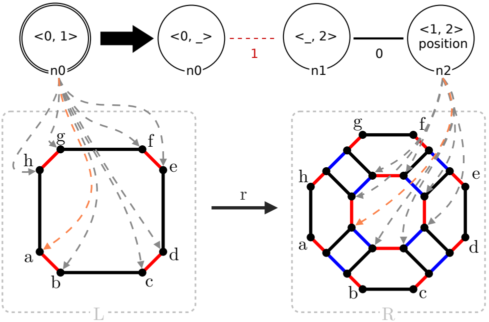


Position of $n2$ as a function of $n0$.

- One equation per dart (8 darts).
- Split per coordinate (on x, y, z).

$$n2.position = \underbrace{w_v\, n0.p_v}_{vertex} + \underbrace{w_e\, n0.p_e}_{edge} + \underbrace{w_f\, n0.p_f}_{face} + \underbrace{w_s\, n0.p_s}_{volume} + \underbrace{w_{cc}\, n0.p_{cc}}_{cc} + t$$

Illustration

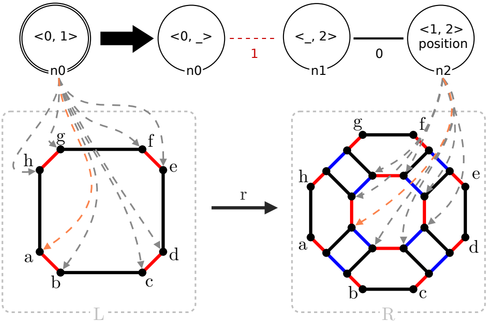


Position of *n*2 as a function of *n*0.

- One equation per dart (8 darts).
- Split per coordinate (on x, y, z).
- 24 equations and 8 variables.

$$n2.position = \underbrace{w_v\, n0.p_v}_{vertex} + \underbrace{w_e\, n0.p_e}_{edge} + \underbrace{w_f\, n0.p_f}_{face} + \underbrace{w_s\, n0.p_s}_{volume} + \underbrace{w_{cc}\, n0.p_{cc}}_{cc} + t$$

Illustration



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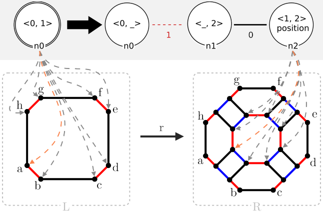
- CSP, solvers used: OR-Tools (Google), Z3 (Microsoft)

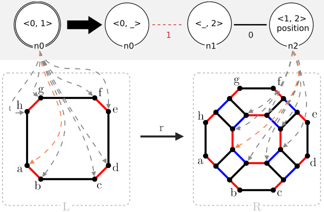
Solving the barycentric triangulation



- Global equation:

$$n2.position = w_v\, n0.p_v + w_e\, n0.p_e + w_f\, n0.p_f + w_s\, n0.p_s + w_{cc}\, n0.p_{cc} + t$$

Solving the barycentric triangulation



- Global equation:

$$n2.position = w_v\, n0.p_v + w_e\, {\color{red} n0.p_e} + w_f\, {\color{blue} n0.p_f} + w_s\, {\color{green} n0.p_s} + w_{cc}\, {\color{orange} n0.p_{cc}} + t$$

- Generated system

$$\begin{cases}
(0.5;0.5) = w_v * (0;0) + w_e * {\color{red}(0.5;0)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\
(0.5;0.5) = w_v * (1;0) + w_e * {\color{red}(0.5;0)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\
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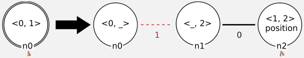
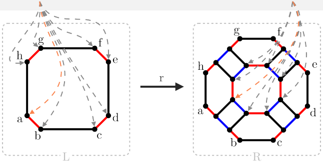
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots
\end{cases}$$

Solving the barycentric triangulation





- Global equation:

$$n2.position = w_v\, n0.p_v + w_e\, {\color{red}n0.p_e} + w_f\, {\color{blue}n0.p_f} + w_s\, {\color{green}n0.p_s} + w_{cc}\, {\color{orange}n0.p_{cc}} + t$$

- Generated system

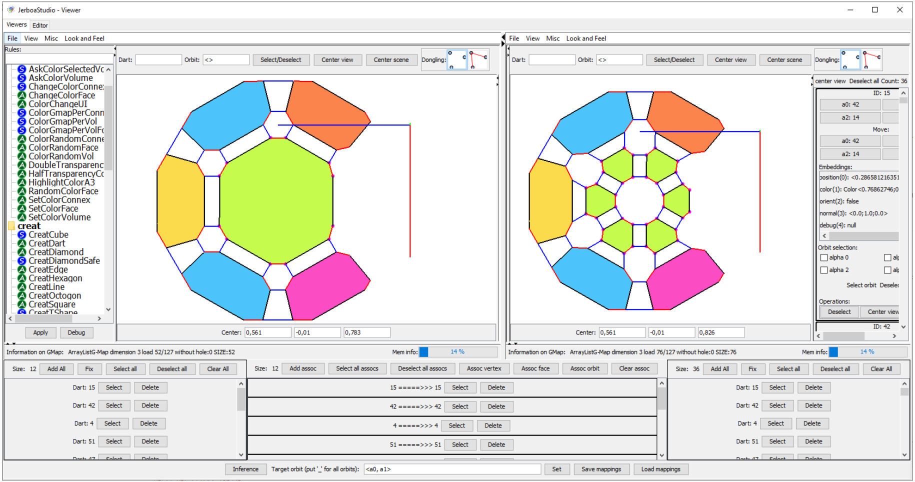
$$\begin{cases} (0.5;0.5) = w_v * (0;0) + w_e * {\color{red}(0.5;0)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\ (0.5;0.5) = w_v * (1;0) + w_e * {\color{red}(0.5;0)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\ (0.5;0.5) = w_v * (1;0) + w_e * {\color{red}(1;0.5)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\ (0.5;0.5) = w_v * (1;1) + w_e * {\color{red}(1;0.5)} + w_f * {\color{blue}(0.5;0.5)} + w_s * {\color{green}(0.5;0.5)} + w_{cc} * {\color{orange}(0.5;0.5)} + (tx;ty) \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{cases}$$

- Solution found:
  - $w_v = 0.0$
  - $w_e = 0.0$
  - $w_f = 1.0$
  - $w_s = 0.0$
  - $w_{cc} = 0.0$
  - $t = (0.0, 0.0)$

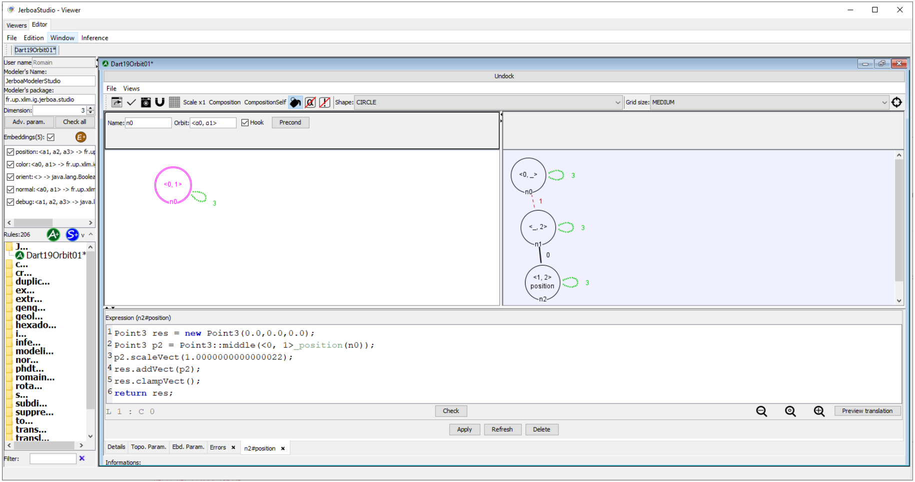
JerboaStudio and applications

- Implementation of the inference mechanism in Jerboa.

JerboaStudio

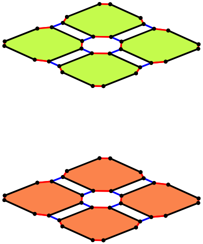


JerboaStudio

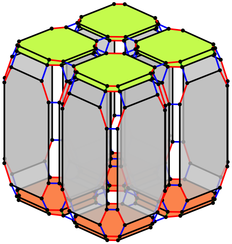


Example inspired from geology

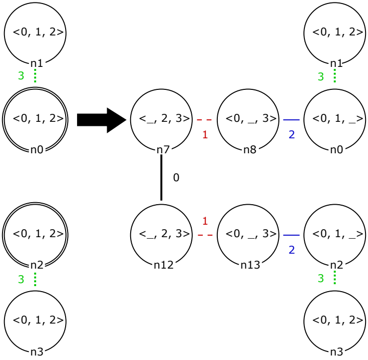
Before



After

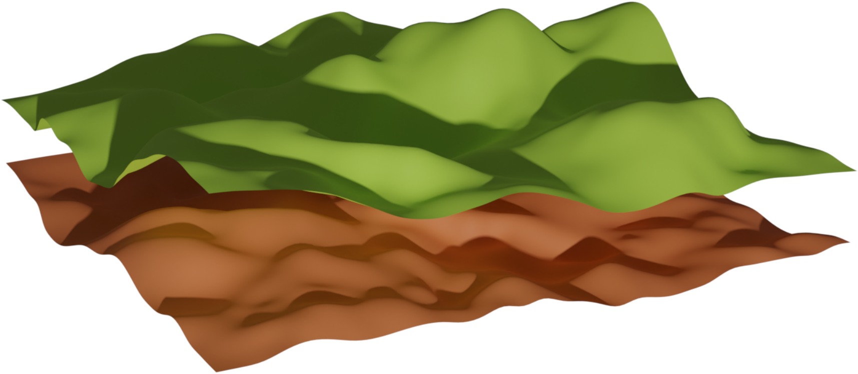


Operation

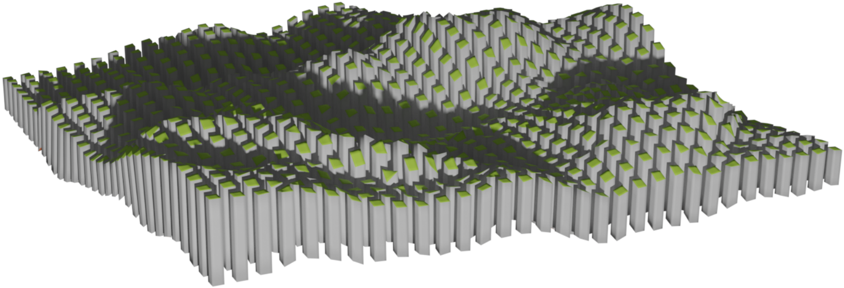


Inference time: ~ 3 ms

Example inspired from geology

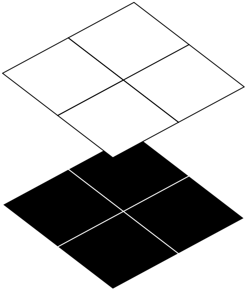


Example inspired from geology

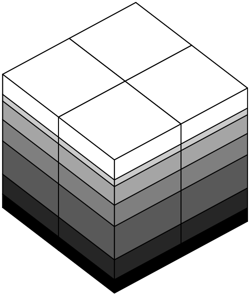


Example inspired from geology (part 2)

Before

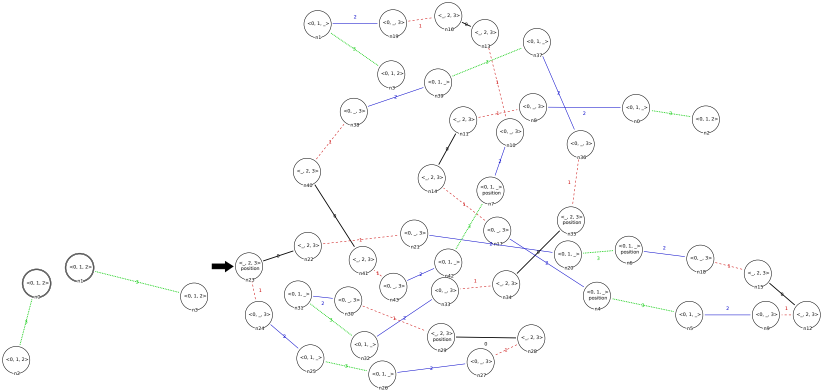


After



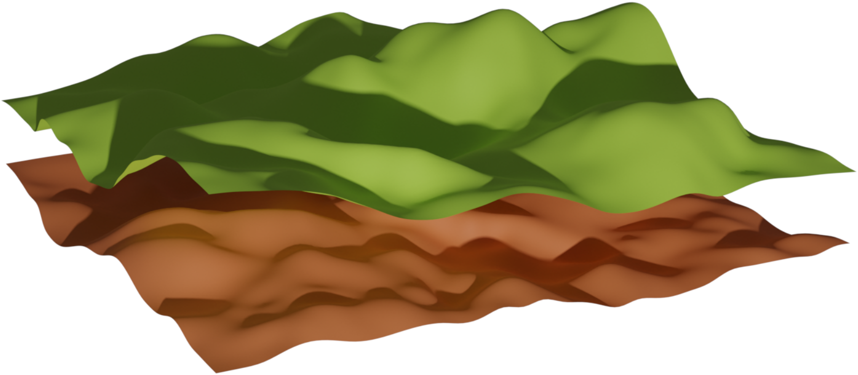
Both positions and colors.

Example inspired from geology (part 2)

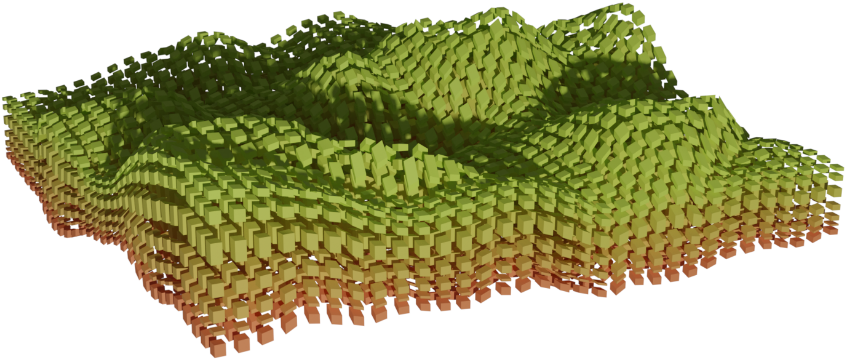


Inference time: $\sim$ 26 ms for the topology,
$\sim$ 549 ms for the embedding expressions

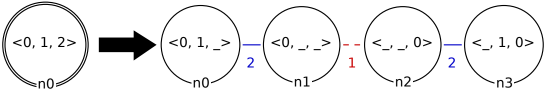
Example inspired from geology

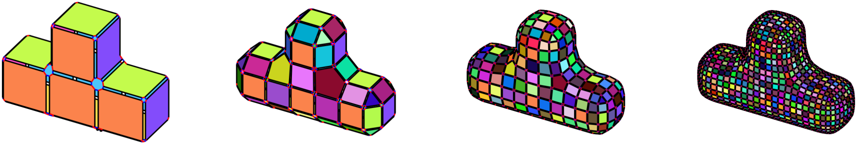


Example inspired from geology

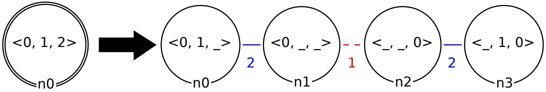


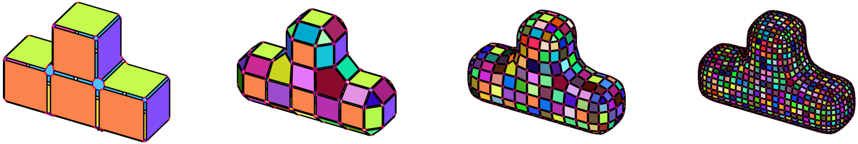
Doo-Sabin subdivision[1]



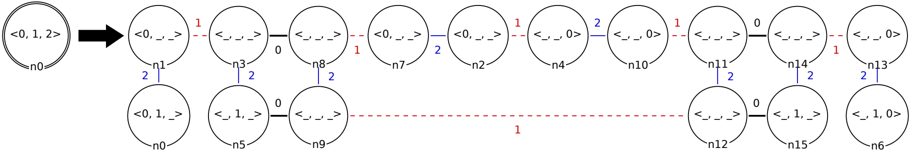


[1] Doo et al. 1978.



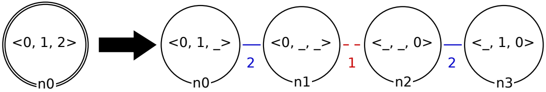


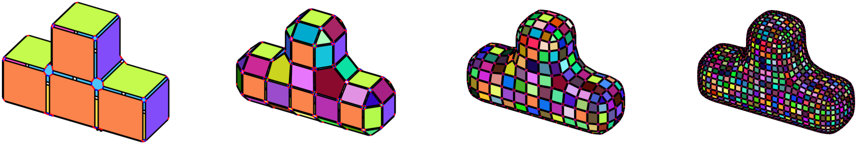
- 2^{nd} iteration:



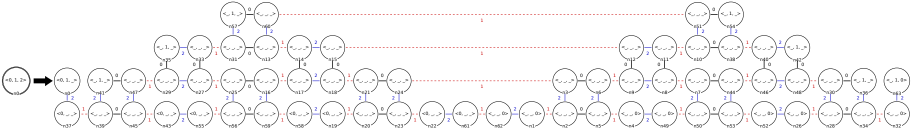
[1]Doo et al. 1978.

Doo-Sabin subdivision[1]



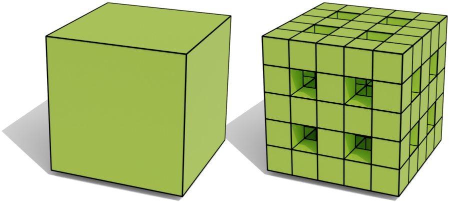


- 3^{rd} iteration:



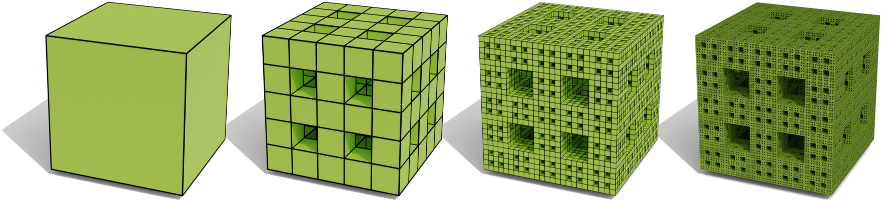
[1]Doo et al. 1978.

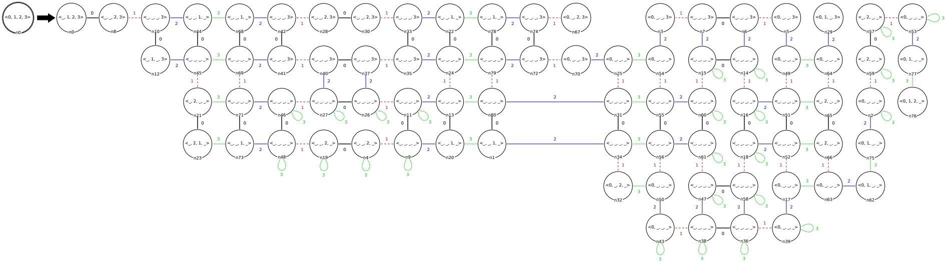
Menger $(2, 2, 2)$[1]



[1] Richaume et al. 2019.

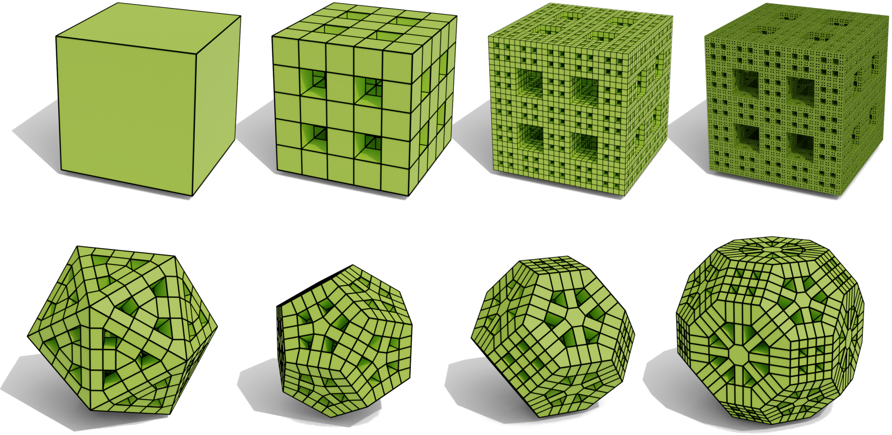
Menger $(2, 2, 2)$[1]





[1]Richaume et al. 2019.

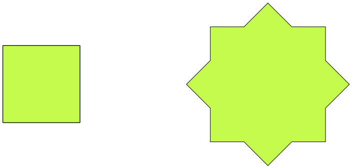
Menger $(2, 2, 2)$[1]



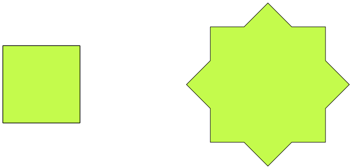
[1]Richaume et al. 2019.

Edge cases

- Von Koch's snowflake generated with L-systems



- Inferred:



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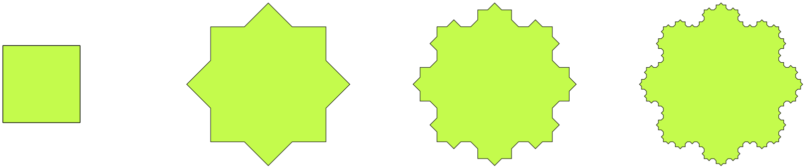
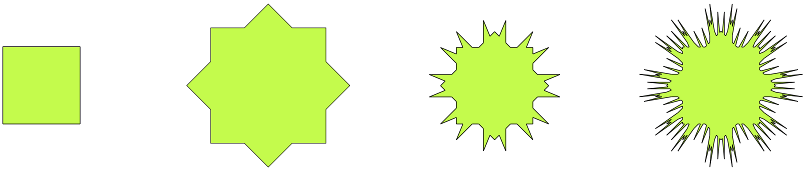
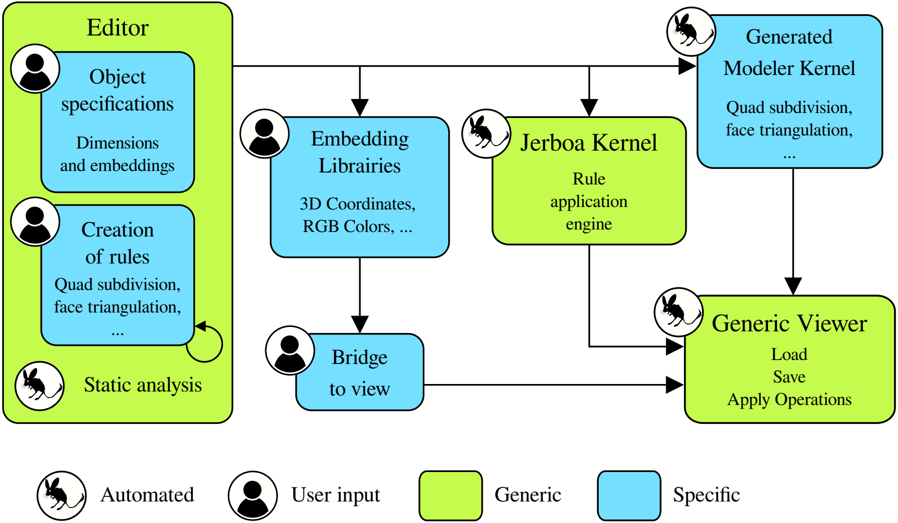
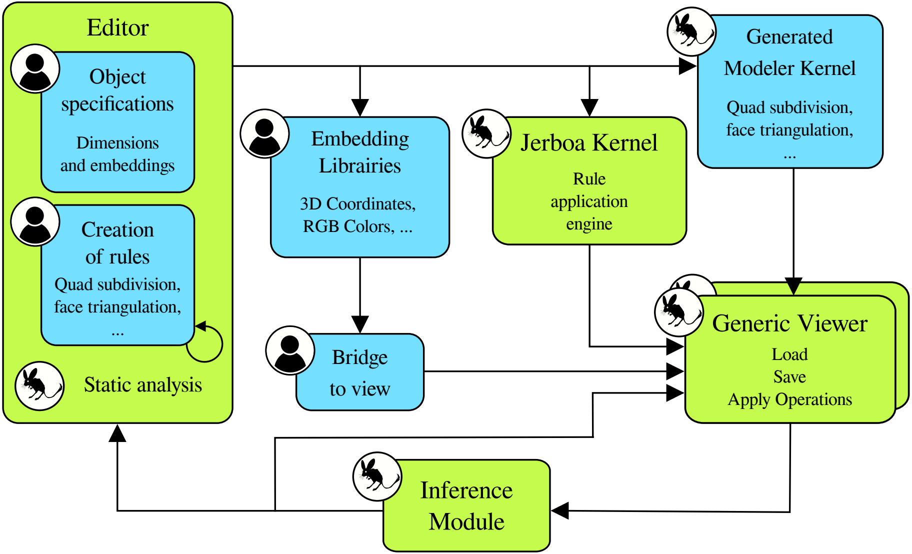


- Inferred:



JerboaStudio's architecture




Editor
Object specifications
Dimensions and embeddings
Creation of rules
Quad subdivision, face triangulation, ...
Static analysis
Embedding Librairies
3D Coordinates, RGB Colors, ...
Bridge to view
Jerboa Kernel
Rule application engine
Generated Modeler Kernel
Quad subdivision, face triangulation, ...
Generic Viewer
Load
Save
Apply Operations
Inference Module

Conclusion

- Ongoing works and main contributions.

Towards local nested conditions?

Rules should be checked statically.

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▶ Collaboration with Nicolas Behr and Pascale Le Gall.

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Towards a multi-cell query-replace approach?

Query-replace modifications with a text editor but for combinatorial structures.[1]

[1] Damiand et al. 2022

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How to extend the approach to multicell patterns?

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How to extend the approach to multicell patterns?

▶ Collaboration with Guillaume Damiand and Vincent Nivoliers supported by the GDR IGRV.

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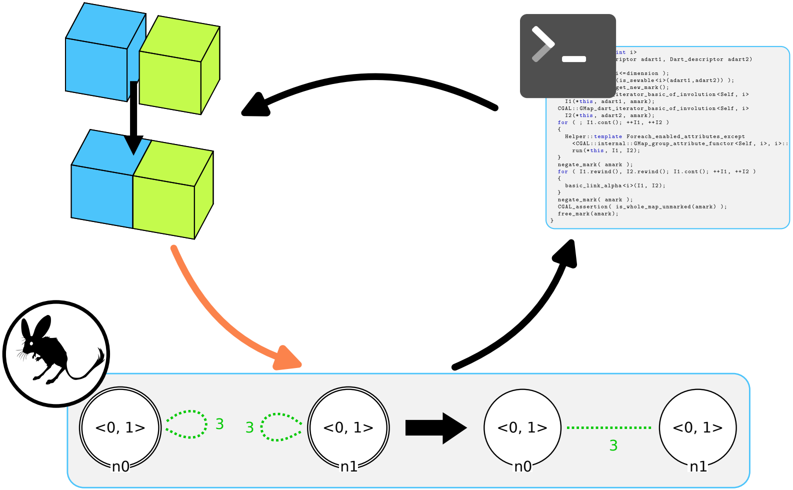
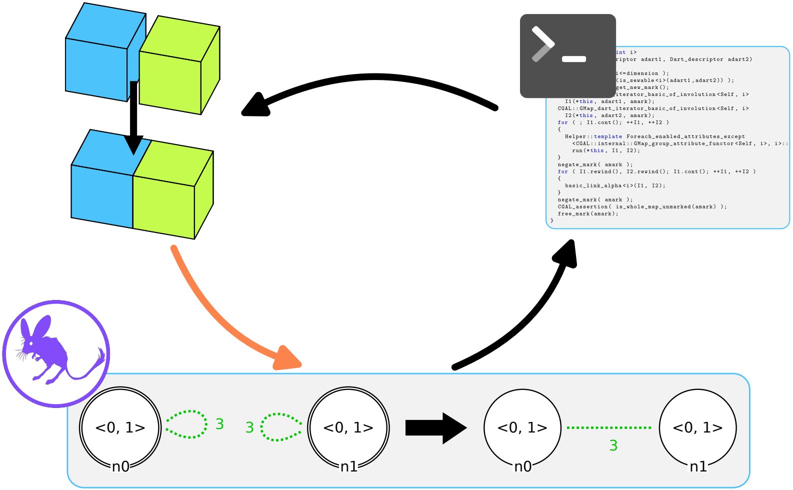
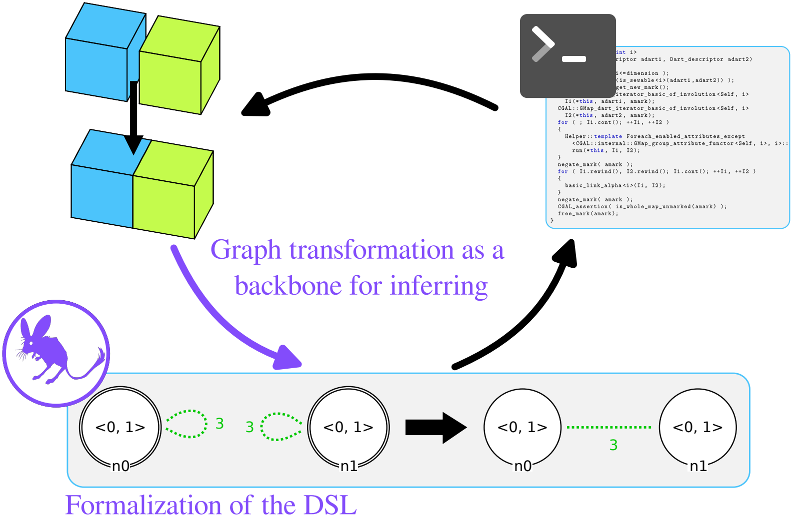
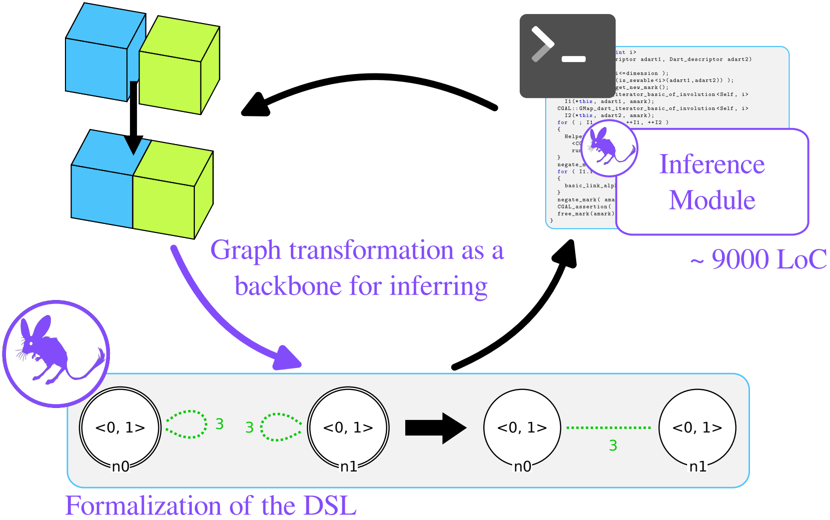

int i>
riptor adart1, Dart_descriptor adart2)
i<=dimension);
(is_sewable<i>(adart1,adart2)));
get_new_mark();
iterator_basic_of_involution<Self, i>
I1(*this, adart1, amark);
CGAL::GMap_dart_iterator_basic_of_involution<Self, i>
I2(*this, adart2, amark);
for (; I1.cont(); ++I1, ++I2)
{
Helper::template Foreach_enabled_attributes_except
<CGAL::internal::GMap_group_attribute_functor<Self, i>, i>::
run(*this, I1, I2);
}
negate_mark(amark);
for (I1.rewind(), I2.rewind(); I1.cont(); ++I1, ++I2)
{
basic_link_alpha<i>(I1, I2);
}
negate_mark(amark);
CGAL_assertion(is_whole_map_unmarked(amark));
free_mark(amark);
}
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1



Formalization of the DSL


Graph transformation as a backbone for inferring
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1
Formalization of the DSL


Inference Module
~ 9000 LoC
Graph transformation as a backbone for inferring
<0, 1>
n0
3
3
<0, 1>
n1
<0, 1>
n0
3
<0, 1>
n1
Formalization of the DSL

Arnould, Agnès et al. (2022). "Preserving consistency in geometric modeling with graph transformations". In: Mathematical Structures in Computer Science. DOI: 10.1017/S0960129522000226.

Belhaouari, Hakim et al. (2014). "Jerboa: A Graph Transformation Library for Topology-Based Geometric Modeling". In: Graph Transformation. ICGT 2014. Ed. by Holger Giese et al. Lecture Notes in Computer Science. Cham: Springer International Publishing, pp. 269–284. ISBN: 978-3-319-09108-2. DOI: 10.1007/978-3-319-09108-2_18.

Bellet, Thomas et al. (2017). "Geometric Modeling: Consistency Preservation Using Two-Layered Variable Substitutions". In: Graph Transformation (ICGT 2017). Ed. by Juan de Lara et al. Vol. 10373. Lecture Notes in Computer Science. Cham: Springer International Publishing, pp. 36–53. ISBN: 978-3-319-61470-0. DOI: 10.1007/978-3-319-61470-0_3.

Damiand, Guillaume et al. (Sept. 19, 2014). Combinatorial Maps: Efficient Data Structures for Computer Graphics and Image Processing. CRC Press. 407 pp. ISBN: 978-1-4822-0652-4.

Damiand, Guillaume et al. (June 18, 2022). "Query-replace operations for topologically controlled 3D mesh editing". In: Computers & Graphics. ISSN: 0097-8493. DOI: 10.1016/j.cag.2022.06.008.

Doo, Daniel et al. (Nov. 1, 1978). "Behaviour of recursive division surfaces near extraordinary points". In: Computer-Aided Design 10.6, pp. 356–360. ISSN: 0010-4485. DOI: 10.1016/0010-4485(78)90111-2.

Ehrig, Hartmut et al. (2006). Fundamentals of Algebraic Graph Transformation. Monographs in Theoretical Computer Science. An EATCS Series. Berlin Heidelberg: Springer-Verlag. ISBN: 978-3-540-31187-4. DOI: 10.1007/3-540-31188-2.

References III

- Heckel, Reiko et al. (2020). Graph Transformation for Software Engineers: With Applications to Model-Based Development and Domain-Specific Language Engineering. Cham: Springer International Publishing. ISBN: 978-3-030-43915-6. DOI: 10.1007/978-3-030-43916-3.
- Pascual, Romain et al. (2022a). "Inferring topological operations on generalized maps: Application to subdivision schemes". In: Graphics and Visual Computing. DOI: 10.1016/j.gvc.2022.200049.
- Pascual, Romain et al. (2022b). "Topological consistency preservation with graph transformation schemes". In: Science of Computer Programming. DOI: 10.1016/j.scico.2021.102728.
- Pennemann, Karl-Heinz (2009). "Development of correct graph transformation systems". PhD thesis. Department für Informatik, Universität Oldenburg, URL: http://oops.uni-oldenburg.de/884.

Poudret, Mathieu et al. (2008). “Graph Transformation for Topology Modelling”. In: Graph Transformations. ICGT 2008. Ed. by Hartmut Ehrig et al. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, pp. 147–161. DOI: 10.1007/978-3-540-87405-8_11.

Richaume, Lydie et al. (2019). “Unfolding Level 1 Menger Polycubes of Arbitrary Size With Help of Outer Faces”. In: Discrete Geometry for Computer Imagery. Ed. by Michel Couprie et al. Lecture Notes in Computer Science. Cham: Springer International Publishing, pp. 457–468. ISBN: 978-3-030-14085-4. DOI: 10.1007/978-3-030-14085-4_36.

Rozenberg, Grzegorz, ed. (Feb. 1, 1997). Handbook of Graph Grammars and Computing by Graph Transformation: Volume I. Foundations. Vol. Foundations. 1 vols. USA: World Scientific Publishing Co., Inc. 545 pp. ISBN: 978-981-02-2884-2.