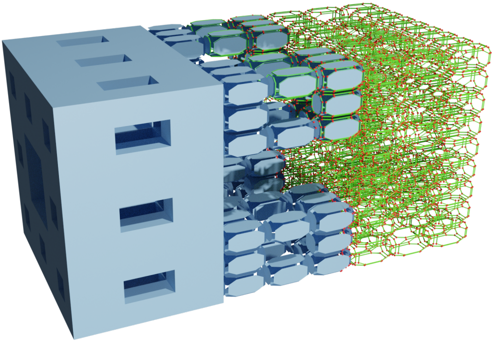
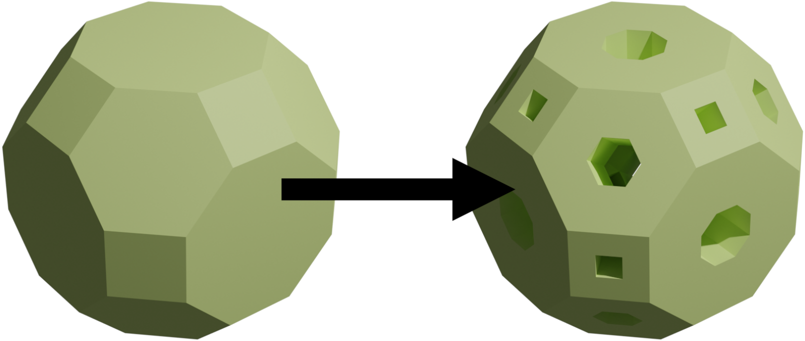
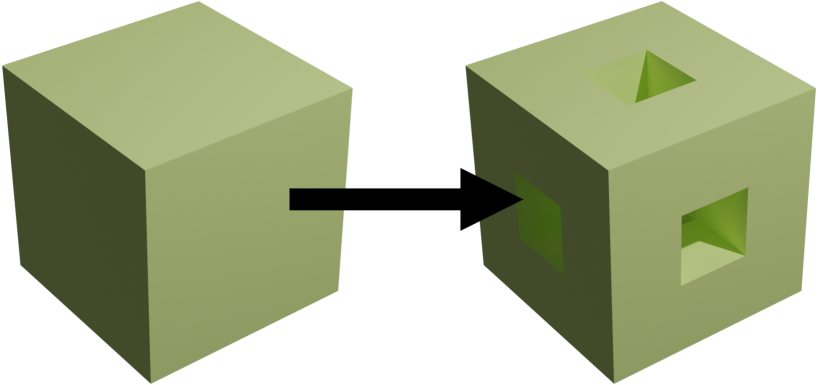
Inference of geometric modeling operations using generalized maps



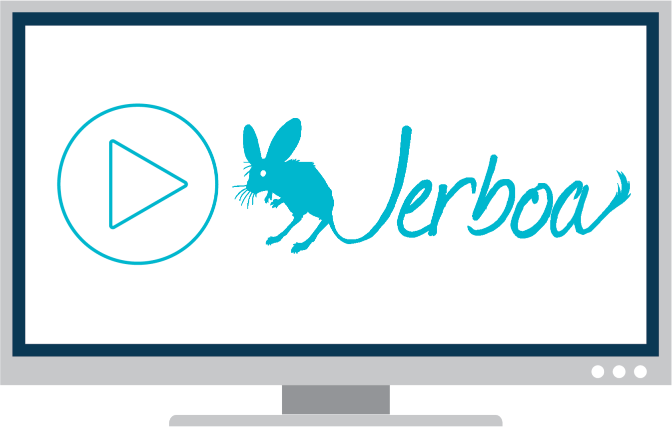
Romain Pascual, Hakim Belhaouari, Agnès Arnould, Pascale Le Gall

June 16th 2022





Demo

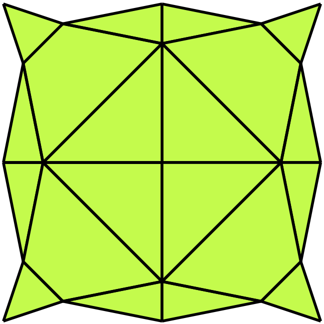


1. Generalized maps
   - Generalized maps
   - Embedded G-maps
2. Graph rewriting
   - Graph Rewriting
   - G-map rewriting
3. Topological inference
   - Main objective
   - Method
   - Examples and results
4. Geometric inference
   - Main objective
   - Method
   - Examples and results

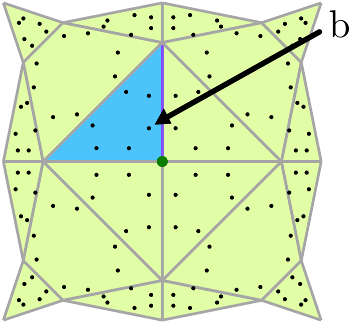
Generalized maps

- Geometric objects are represented with embedded generalize maps.

Generalized maps [Damiand and Lienhardt 2004]



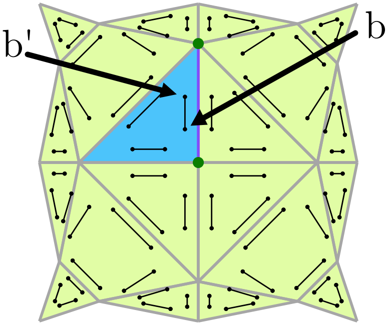
Generalized maps [Damiand and Lienhardt 2004]



G-maps built as graphs.

b identifies the green vertex, the purple edge and the blue face.

Generalized maps [Damiand and Lienhardt 2004]

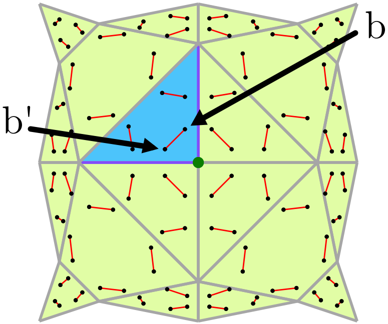


G-maps built as graphs.

0-arc :

- distinct vertices.
- same edges and faces.

Generalized maps [Damiand and Lienhardt 2004]

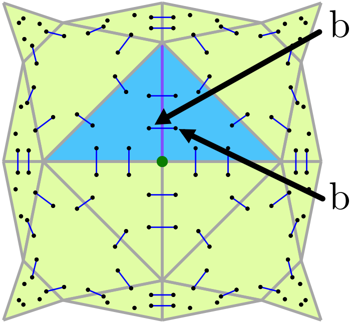


G-maps built as graphs.

1-arc :

- distinct edges.
- same vertices and faces.

Generalized maps [Damiand and Lienhardt 2004]

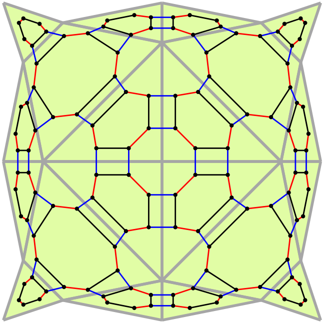


G-maps built as graphs.

2-arc :

- distinct faces.
- same vertices and edges.

Generalized maps [Damiand and Lienhardt 2004]

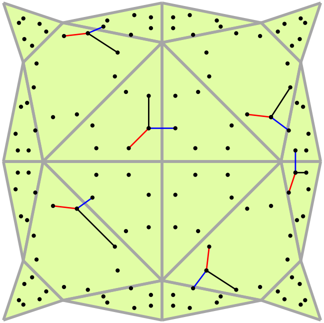


G-maps built as graphs.

n-G-map (union of the graphs)

Undirected graph labeled on the arcs with dimensions from $[\![0, n]\!]$ such that :

Generalized maps [Damiand and Lienhardt 2004]



G-maps built as graphs.

n-G-map (union of the graphs)

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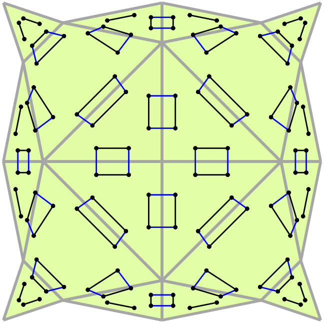
- Incidence : every dart (node) if the source of a unique arc per dimension.

Generalized maps [Damiand and Lienhardt 2004]



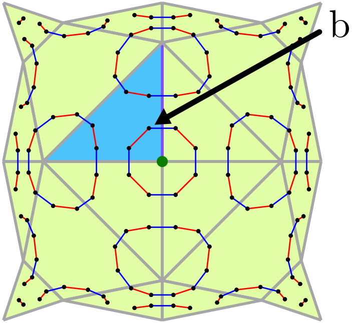
G-maps built as graphs.

n-G-map (union of the graphs)

Undirected graph labeled on the arcs with dimensions from $[\![0, n]\!]$ such that :

- Incidence : every dart (node) if the source of a unique arc per dimension.
- Cycles : any $ijij$-path is a cycle whenever $i + 2 \leq j$

Topological cells and orbits

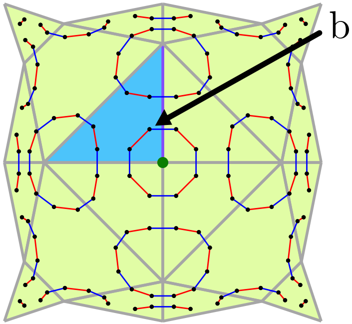


Topological cells correspond to subgraphs.

Orbit

Graph induced by a subset $\langle o \rangle \subseteq [\![0, n]\!]$ of dimensions.

Topological cells and orbits

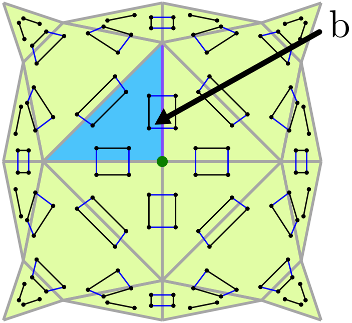


Topological cells correspond to subgraphs.

b belongs to the green vertex.

- $\langle 1, 2 \rangle$-orbit .

Topological cells and orbits

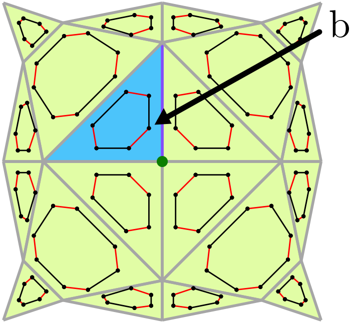


Topological cells correspond to subgraphs.

b belongs to the purple edge.

- $\langle 0, 2 \rangle$-orbit .

Topological cells and orbits



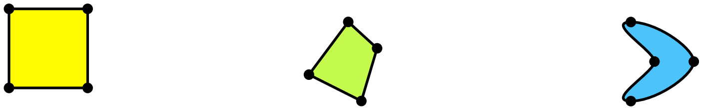
Topological cells correspond to subgraphs.

b belongs to the blue face.

- $\langle 0, 1 \rangle$-orbit .

Geometry

- An object's display rely on it's geometry.



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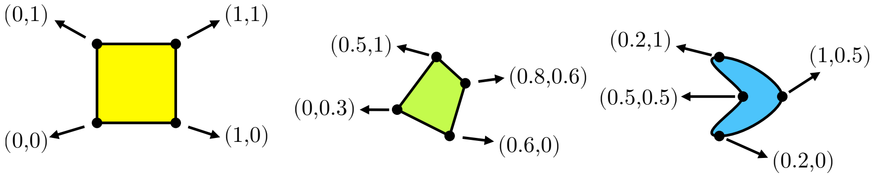

An embedding is defined for an orbit type : position.

Geometry

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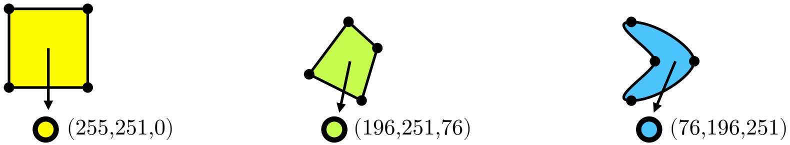
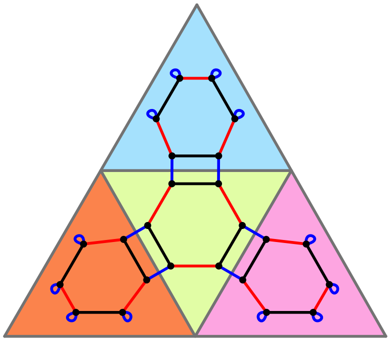


An embedding is defined for an orbit type : color.

Embedded G-maps

▶ Embedding values are defined for each dart, with some consistency constraints.



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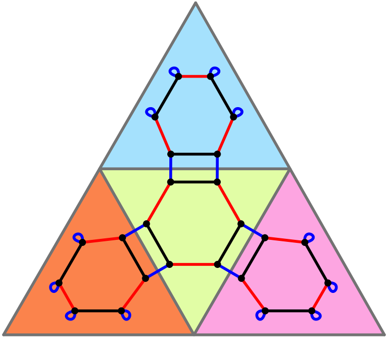
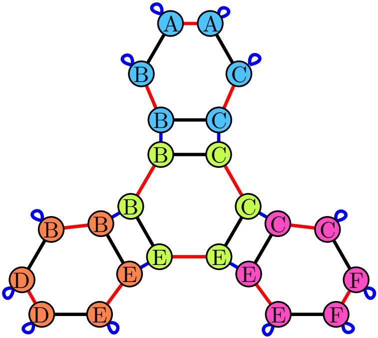
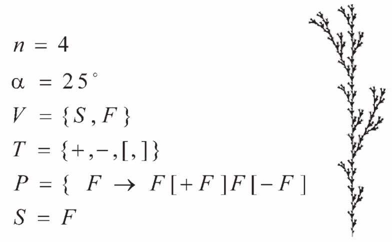




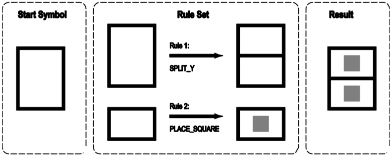
Graph rewriting

- Operations on G-maps are designed as graph rewriting rules.

Rewriting



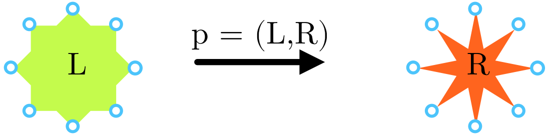
L-systems
([Santos et Coelho 2009])



Shape grammars
([Di Angelo et al. 2012])

Graph transformation rules

- Goal : Generalise from strings to graphs.



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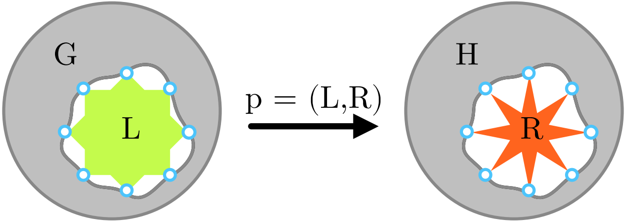
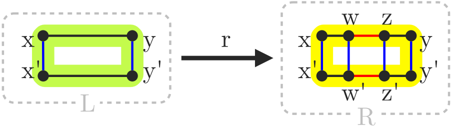
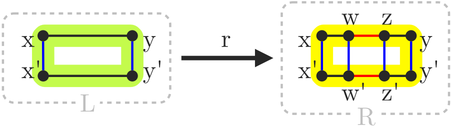


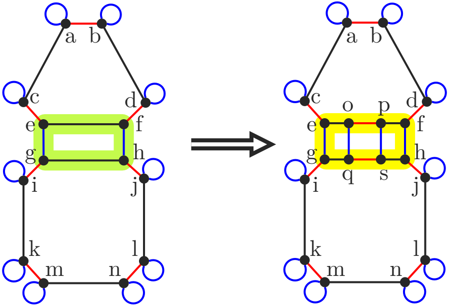
- Find in G a graph similar to L,
- Remove it from G,
- Reconnect R within the more global context.

G-map rewriting : vertex insertion

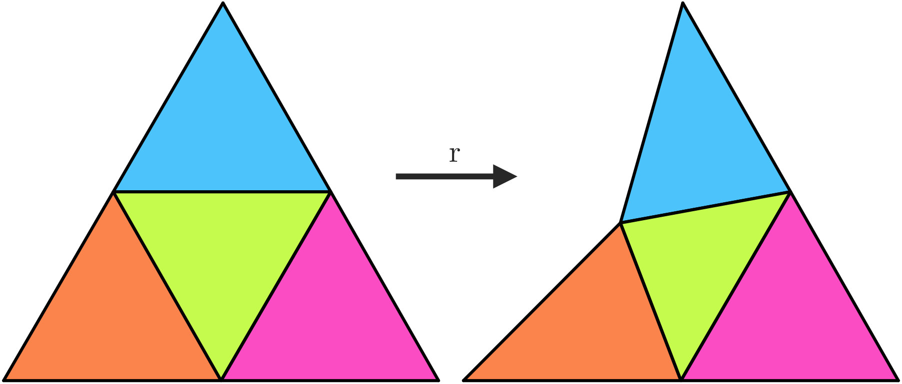


G-map rewriting : vertex insertion





Embedding expressions : vertex translation



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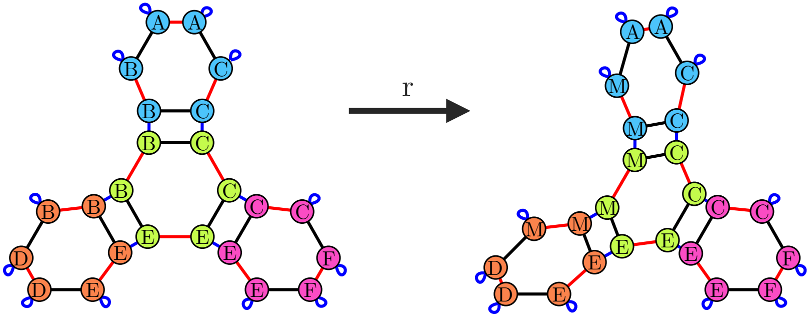

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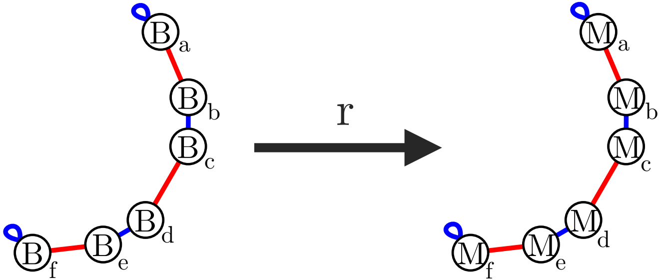

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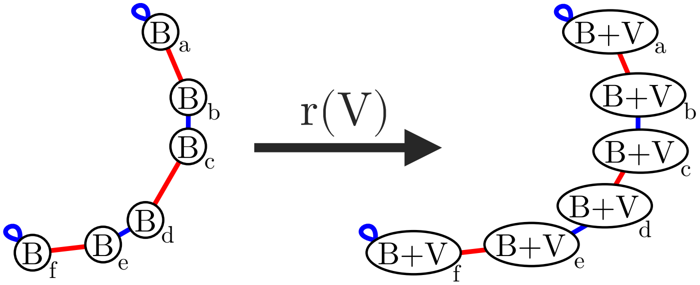
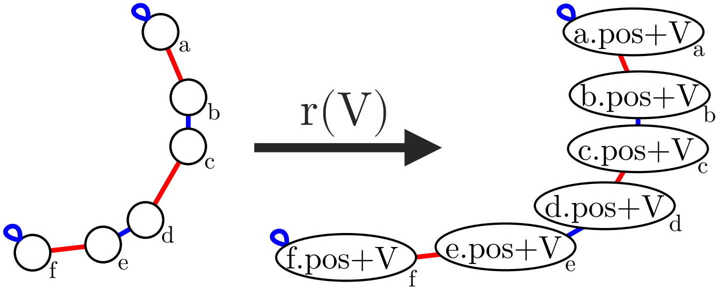


- Computations in the embedding space
  - $B + V = M$

Embedding expressions : vertex translation

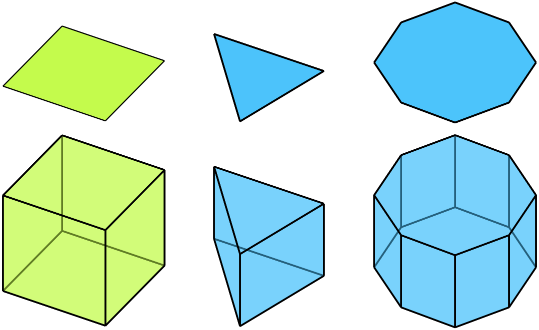


- Computations in the embedding space
- Value accessors
  - $a.position = B$

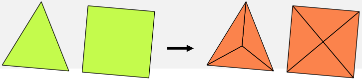
Need for genericity (1) : Topology

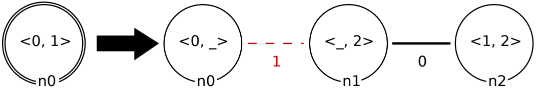
Exploit the homogeneity of G-maps.

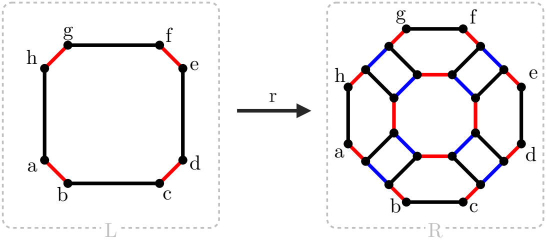
- Key idea : modeling operation are parameterized by the topology (more precisely orbits).



Orbit rewriting







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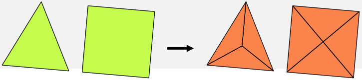

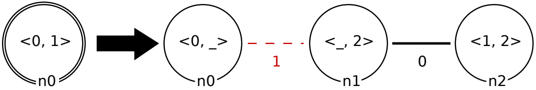

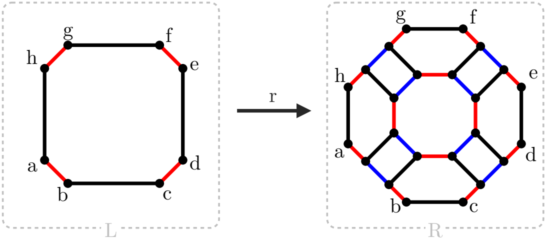

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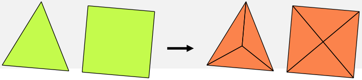

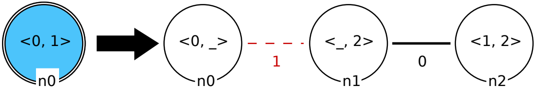

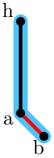

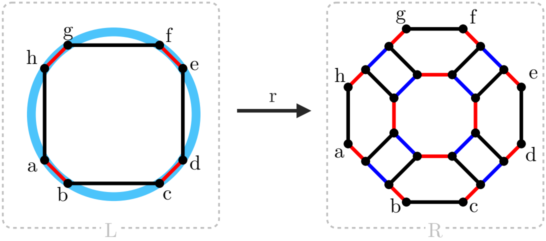

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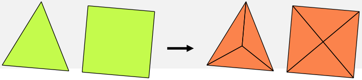

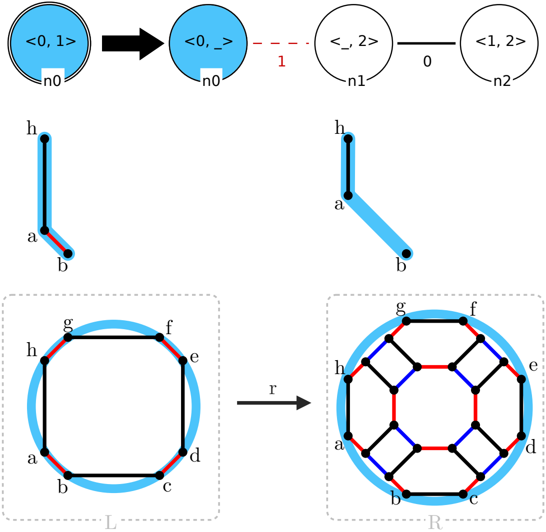

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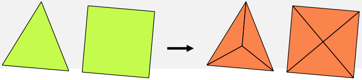

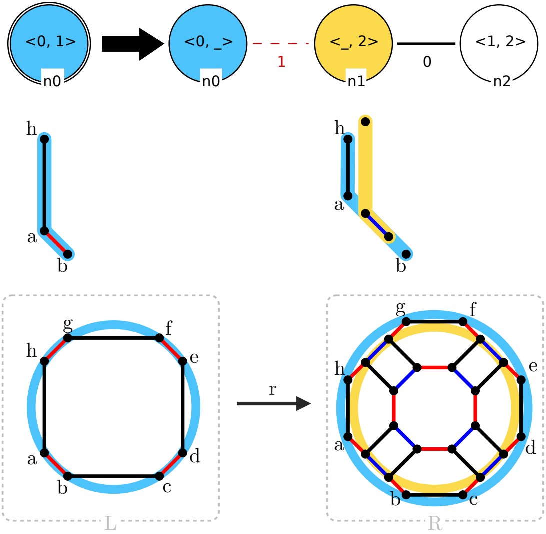

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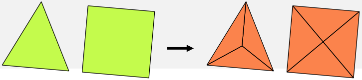

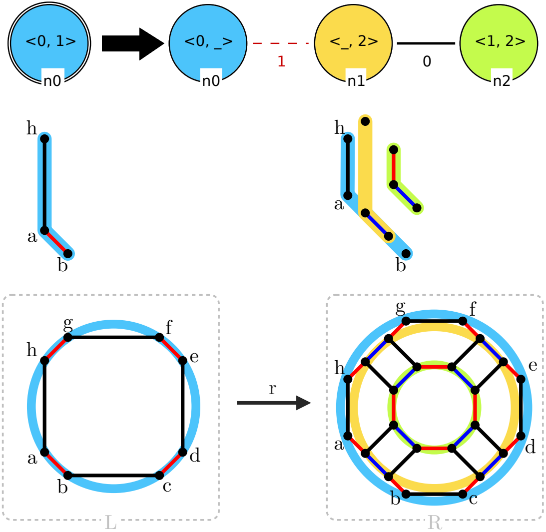

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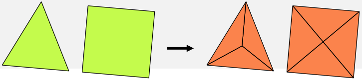

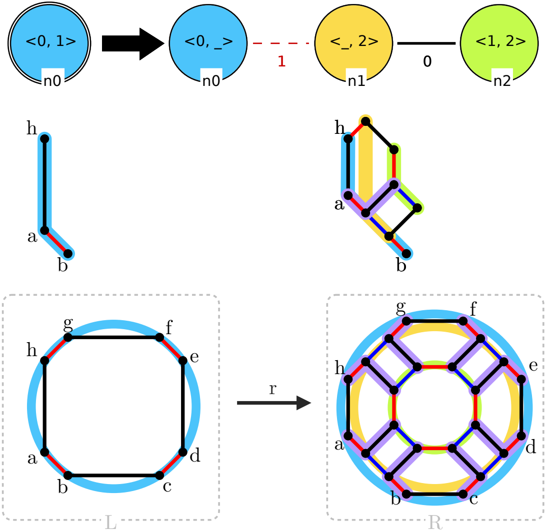

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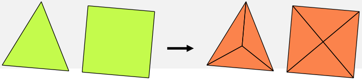

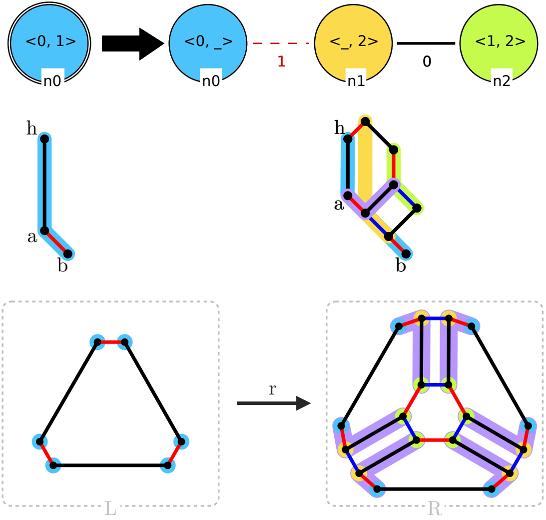

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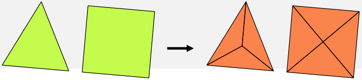
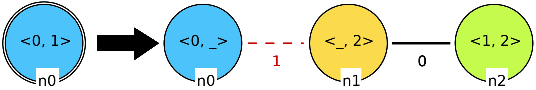
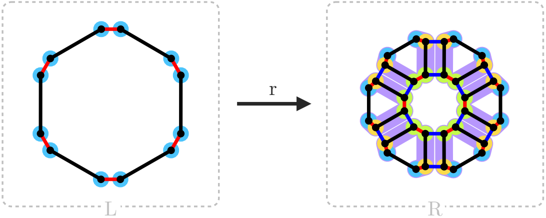
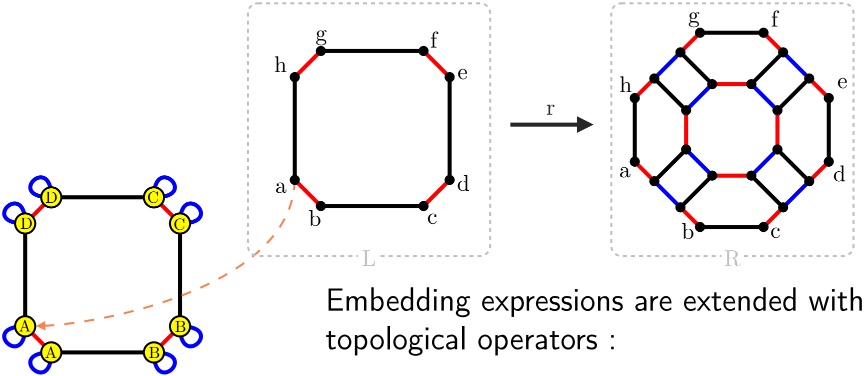






Need for genericity (2) : Geometry



Embedding expressions are extended with topological operators :

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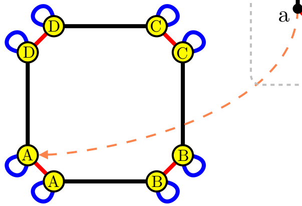

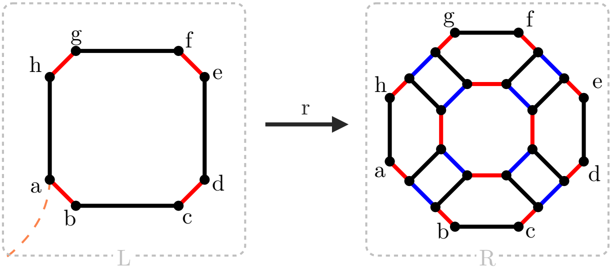

Embedding expressions are extended with topological operators :

- Neighbor operator :
  - a@0.position = D
  - a@0@1@0.position = C

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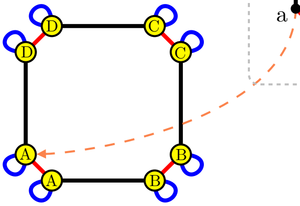
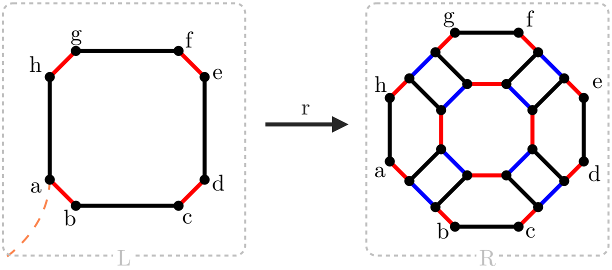
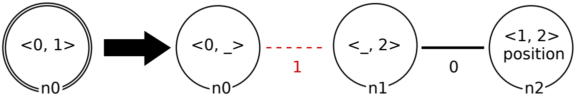


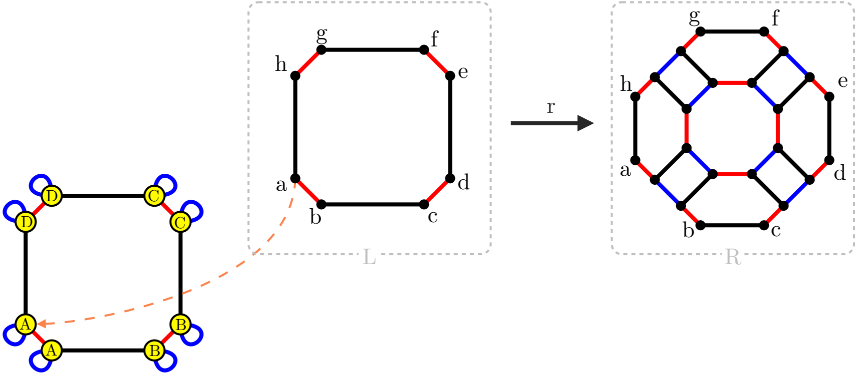


Embedding expressions are extended with topological operators :

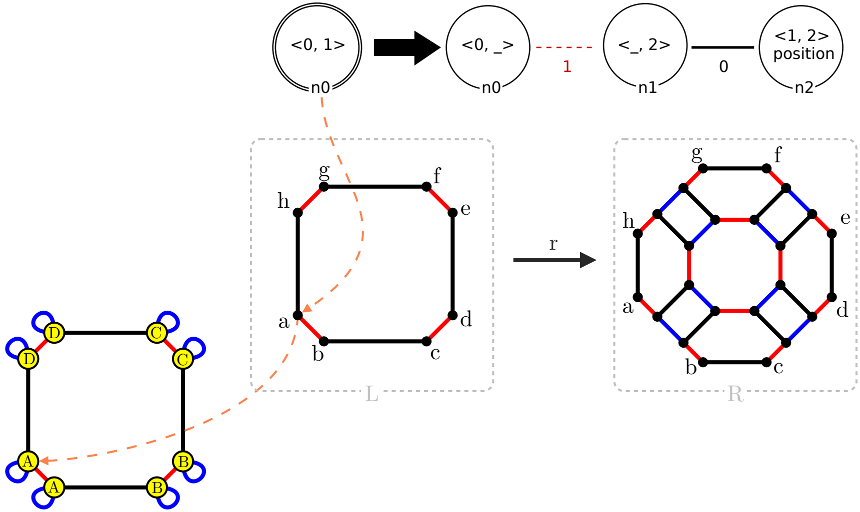
- Neighbor operator :
- Collect operator :
  - $position_{\langle 0,2 \rangle}(a) = \{A, D\}$
  - $position_{\langle 0,1 \rangle}(a) = \{A, B, C, D\}$

Extension to schemes

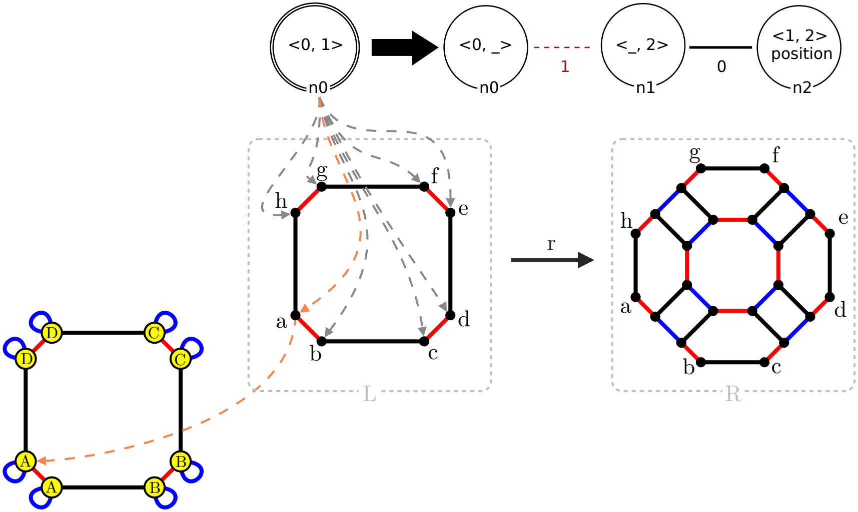




Extension to schemes



Extension to schemes



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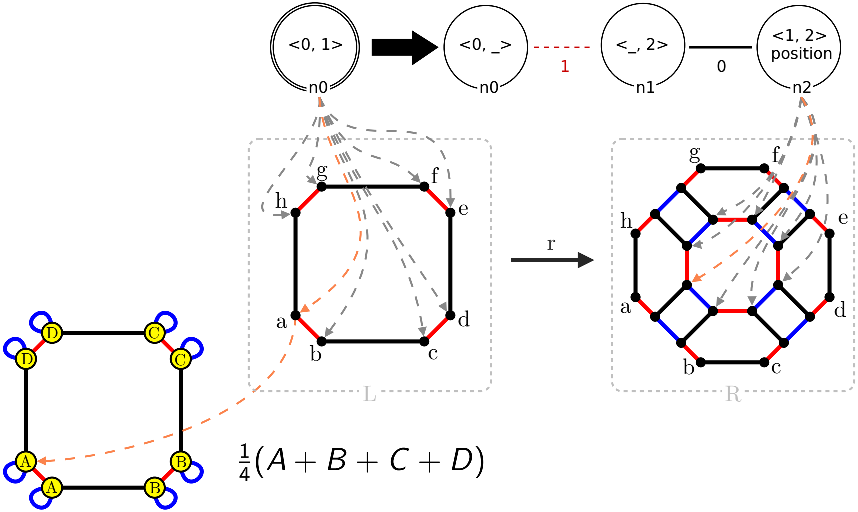
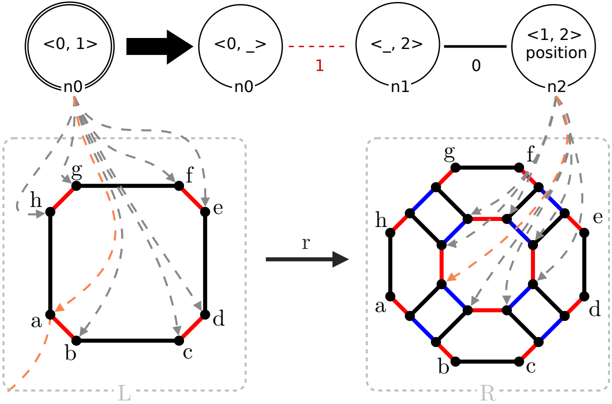
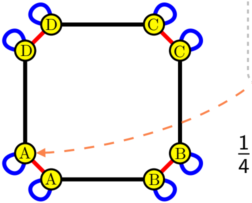


$\frac{1}{4}(A + B + C + D)$

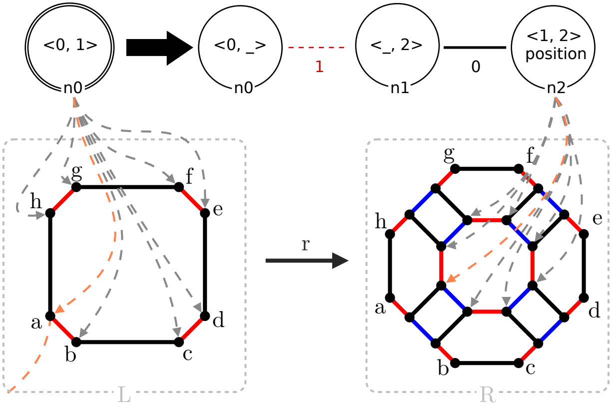
Extension to schemes

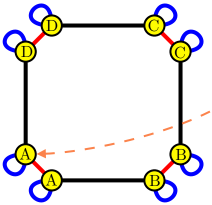




$$\frac{1}{4}(A + B + C + D) = middle(\{A, B, C, D\})$$

Extension to schemes





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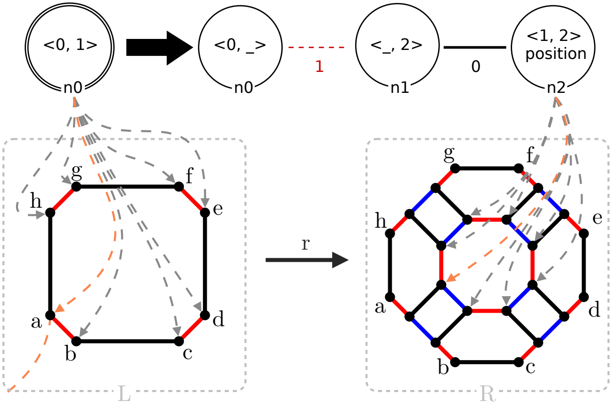
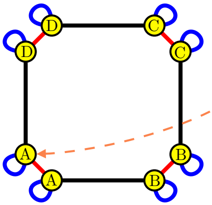
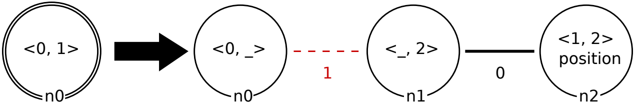




$$\frac{1}{4}(A+B+C+D) = middle(\{A,B,C,D\})$$
$$= middle(position_{\langle 0,1\rangle}(a))$$
$$= middle(position_{\langle 0,1\rangle}(n0))$$

Key points

We describe geometric modeling operations with rule schemes.



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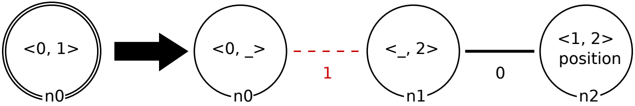

- Operations described in a domain-specific language

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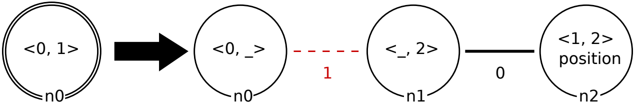

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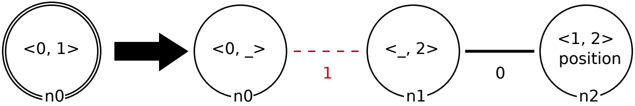


- Operations described in a domain-specific language
- Each node in a pattern of a rule encodes for multiple darts in the G-maps.
- Node names in the embedding expressions are substituted during the instantiation.

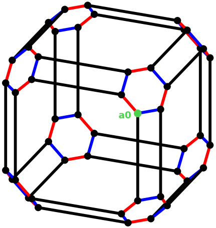
Topological inference

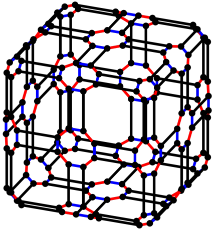
- Graph traversal algorithm to infer the topological part of modeling operations.

Objective

Retrieve a scheme rule from an instance of an object before modification and one after modification.

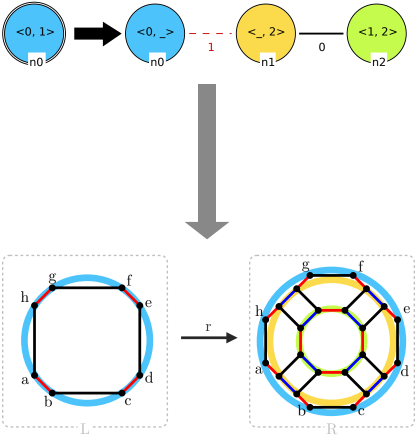
- Input : Before G-map, After G-map and some additional information.
- Output : Rule scheme(s) describing the modeling operation.





- Dart $a0$
- Orbit type $\langle 1,\ 2 \rangle$

Reversing the instantiation process



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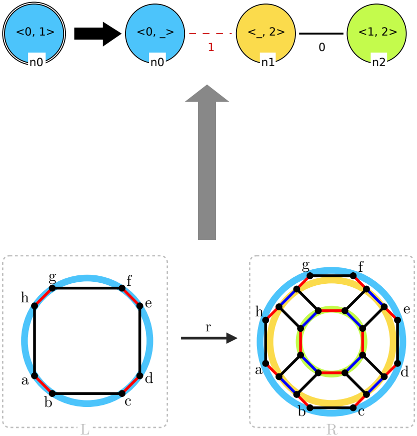


Algorithm : Topological folding algorithm

Input: A graph G encoding the preservation relation between two partial G-maps, an orbit type $\langle o \rangle$, and a dart a of G.

Output: A graph $\mathcal{S}$ that encodes the Jerboa rule with $\langle o \rangle$ as variable, given that the operation is applied at the dart a.

$Q \leftarrow \emptyset,\ \mathcal{S} \leftarrow \emptyset$ // empty queue and empty ‘rule‘ graph
$h \leftarrow Node(G, \langle o \rangle, a)$ // build the hook node
$add\_node(\mathcal{S}, h)$ // add h to the ‘rule‘ graph
$enqueue(Q, h)$
while $Q \neq \emptyset$ **do**
  $m \leftarrow dequeue(Q)$
  foreach $d \in [\![0, n]\!] \setminus label(m)$ **do**
    $v \leftarrow arc\_expansion(G, m, d)$ // extend arcs
    $build\_label(G, v)$ // deduce the relabeling function
    $add\_node(\mathcal{S}, v)$
    $enqueue(Q, v)$
return S

R. Pascual, H. Belhaouari, A. Arnould, and P. Le Gall, ‘Inferring topological operations on generalized maps : Application to subdivision schemes’, Graphics and Visual Computing, 2022.

Folding a G-map

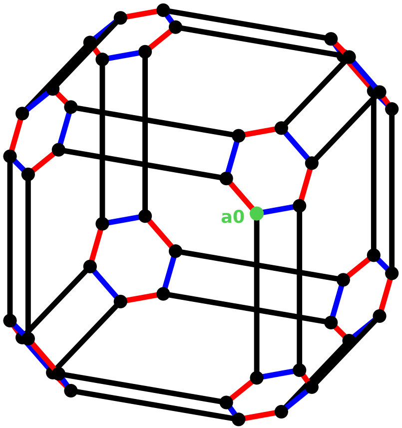
Besides a G-map, we whose a dart in the G-map and an orbit type.

Graph traversal algorithm

Iteratively apply two foldings :

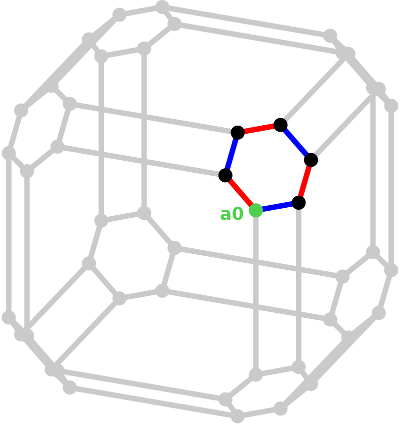
- Folding of a node.
- Folding of the arcs.

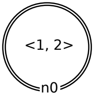
▶ Illustration on the cube with the orbit type $\langle 1, 2 \rangle$.



Execution

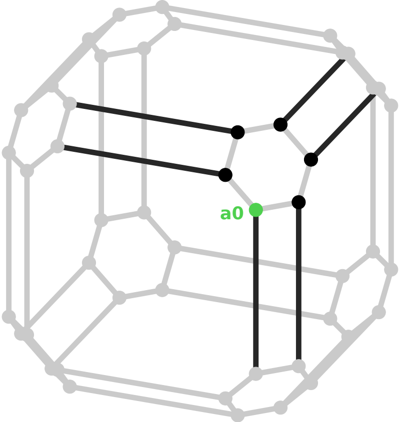
Folding of a node (hook case with the orbit type $\langle 1, 2 \rangle$).

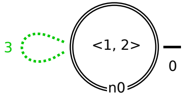




Execution

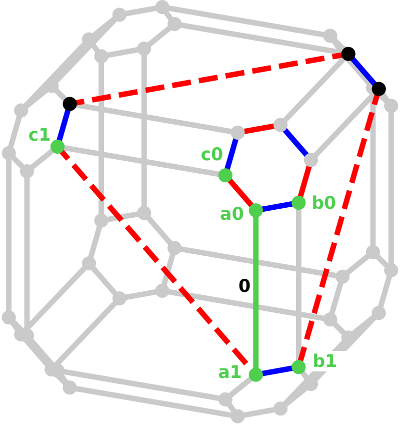
Folding of the arcs.

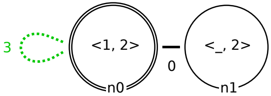




Execution

Folding of a node.





Execution

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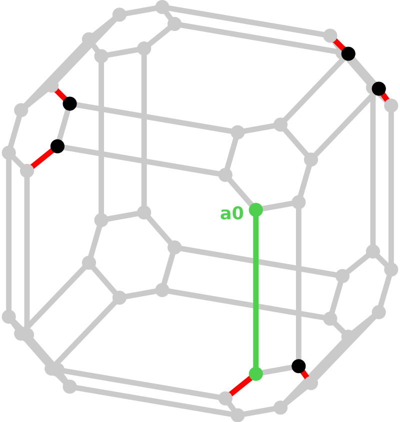
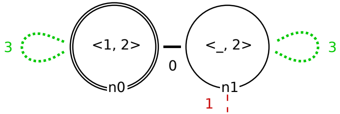
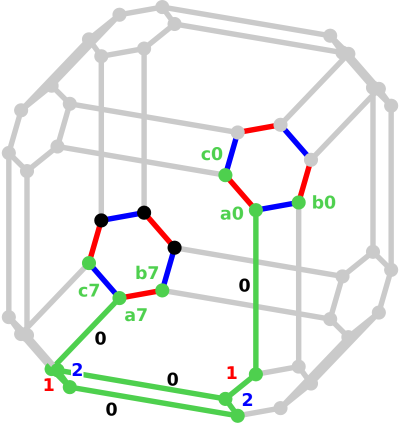
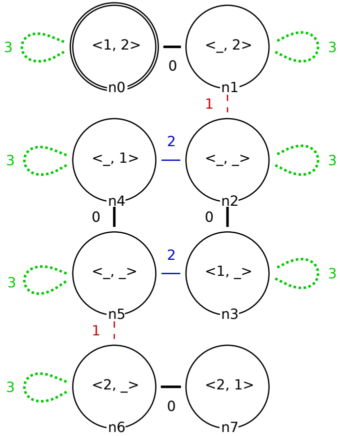




Execution

Folding of a node.





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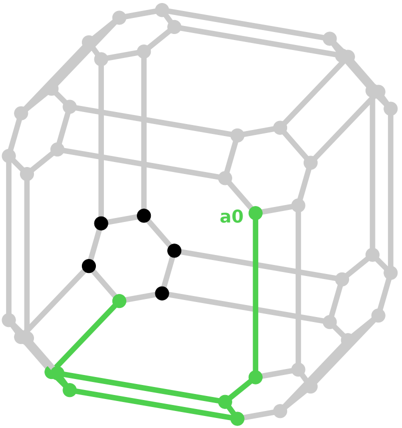
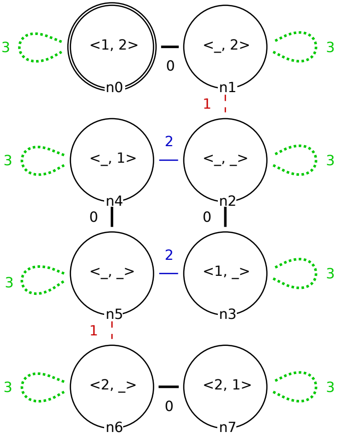
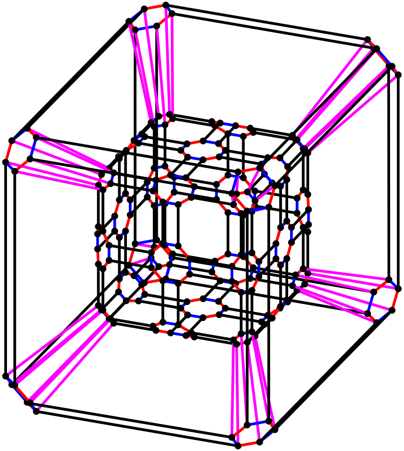




Folding of a rule

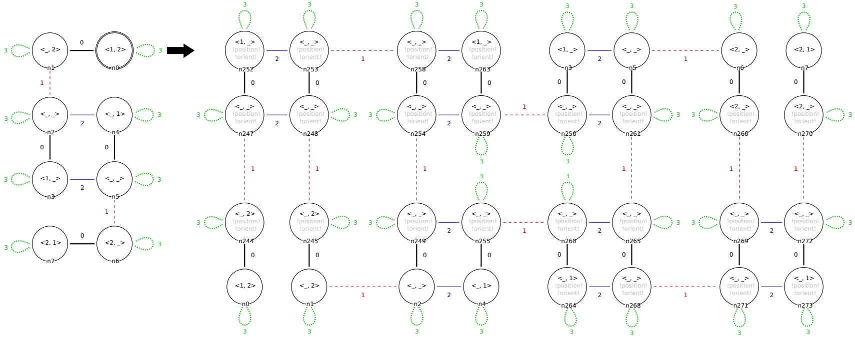
Partial mapping on darts from the before instance to the after instance.

Build a graph where the preserved darts are linked with κ-arcs (in pink).



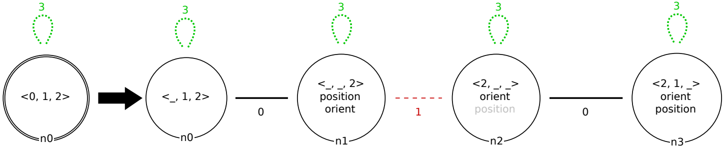
Folding the quad subdivision

- Rule scheme with the orbit type $\langle 1, 2 \rangle$ on the cube :

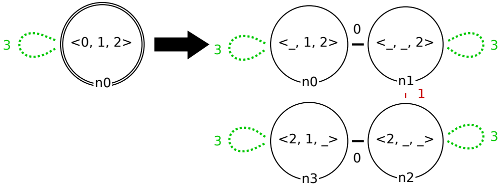


Folding the quad subdivision

- Used rule scheme :



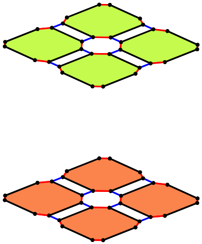
- Among the inferred rules, we retrieve the one we used.



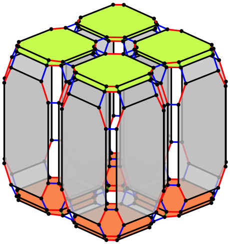
  - 768 possible schemes
  - 14 distinct schemes (cube symmetry).
  - 48 schemes tried (marking).
  - 14 schemes built (removal of isomorphic rules).

Example in geology (pure topology)

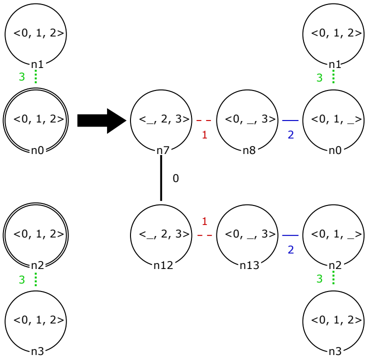
Before



After



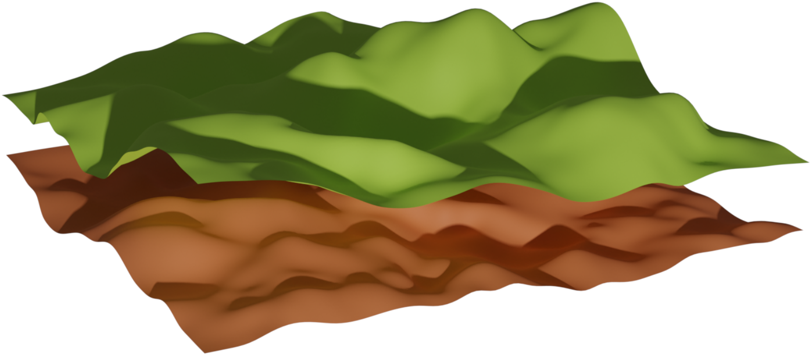
Operation



Inference time : ~ 3 ms

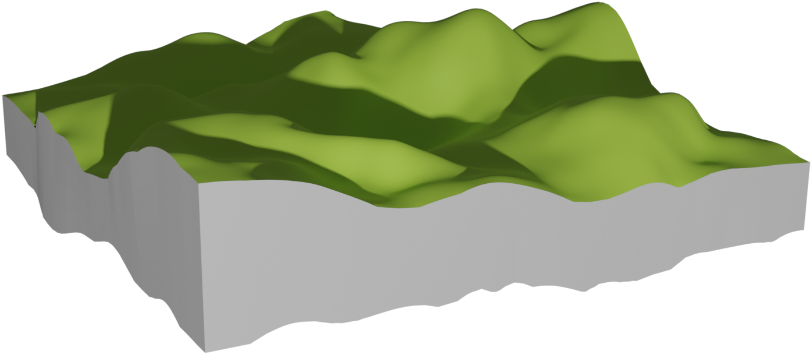
Example in geology (pure topology)

Before



Example in geology (pure topology)

After



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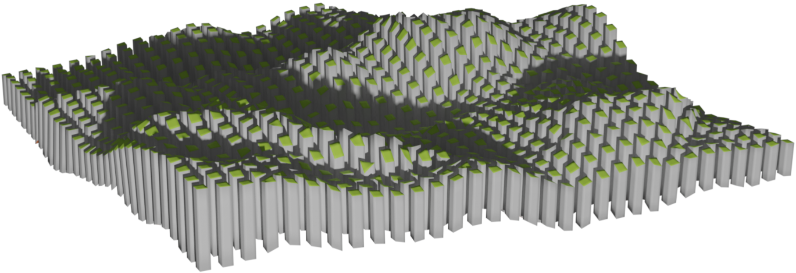
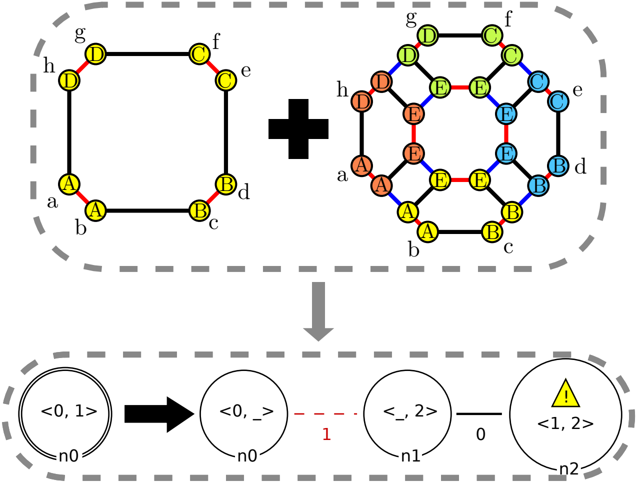


Geometric inference

- Computing embedding expressions is solved as a constraint solving problem.

Objective



The rule is missing its embedding expressions.

Method (inference of positions)

▶ Hypothesis : The vertex positions of the target object C are obtained as affine combinations of vertex positions in the initial object O.

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For each vertex in C, we want a position p expressed as :

$$p = \sum_{i=0}^{k} a_i p_i + t$$

where :

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- t : translation (unknown)

Need for abstraction on schemes

We want $(a_i)_{0 \leq i \leq k}$ such that :
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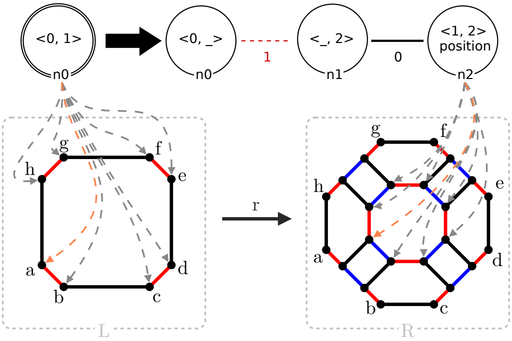

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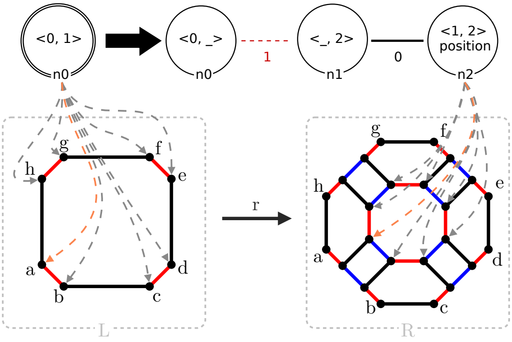


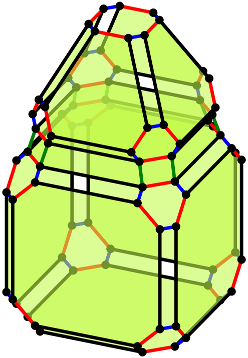
Issue : darts in the G-map will share the same expression.

- Because rule schemes abstract topological cells.

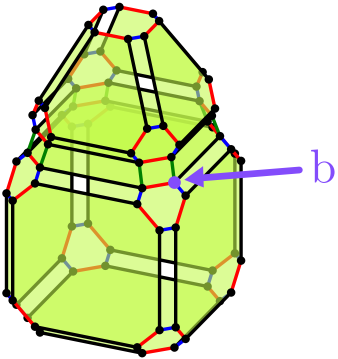
Solution : Exploit the topology.

- Use points of interests that share the same expression.

Points of interests



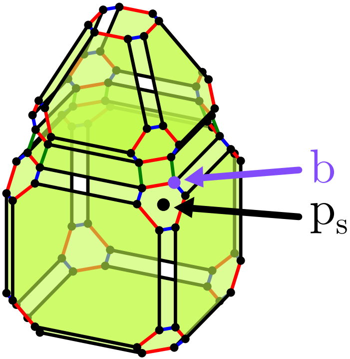
Points of interests



Points of interests

avec

- p_s : vertex

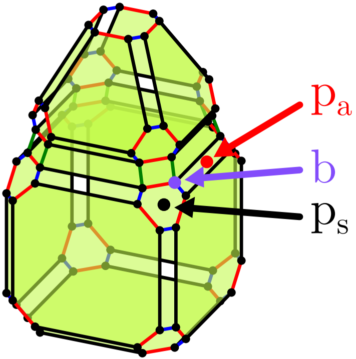


$$p_s = middle(position_{\langle 1,2,3 \rangle}(b))$$

Points of interests

avec

- p_s : vertex
- p_a : edge midpoint

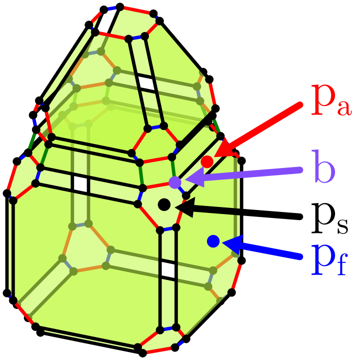


$$p_a = middle(position_{\langle 0,2,3 \rangle}(b))$$

Points of interests

avec

- p_s : vertex
- p_a : edge midpoint
- p_f : face barycenter

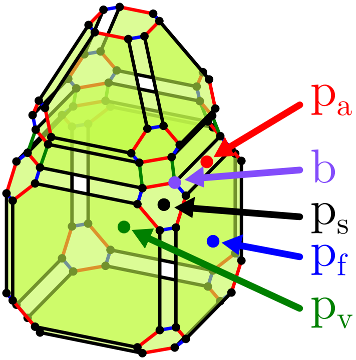


$$p_f = middle(position_{\langle 0,1,3 \rangle}(b))$$

Points of interests

avec

- p_s : vertex
- p_a : edge midpoint
- p_f : face barycenter
- p_v : volume barycenter

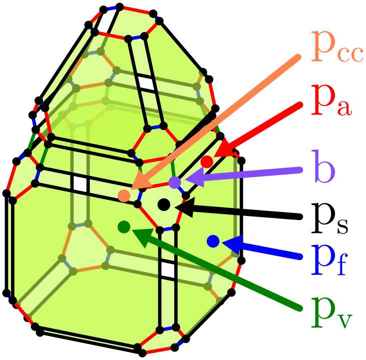


$$p_v = middle(position_{\langle 0,1,2\rangle}(b))$$

Points of interests

avec

- p_s : vertex
- p_a : edge midpoint
- p_f : face barycenter
- p_v : volume barycenter
- p_{cc} : CC barycenter



$$p_{cc} = middle(position_{\langle 0,1,2,3\rangle}(b))$$

Points of interests

avec

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- p_a : edge midpoint
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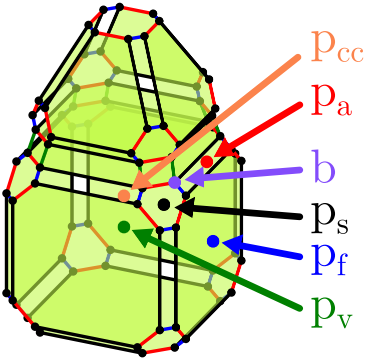
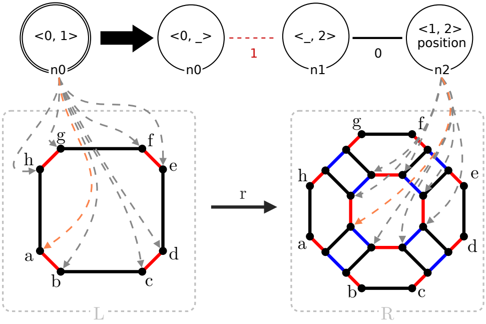


Thanks to the points of interests, the systems is rewritten as :

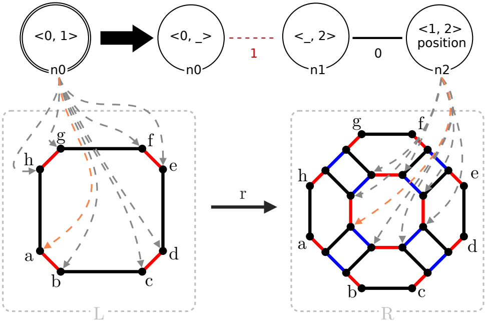
$$p = a_s p_s + a_a p_a + a_f p_f + a_v p_v + a_{cc} p_{cc} + t$$

Illustration



The position expression of $n2$ only depends of $n0$.

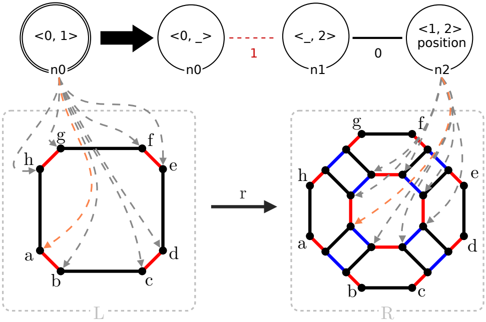
Illustration



The position expression of $n2$ only depends of $n0$.

$$n2.position = a_s n0.p_s + a_a n0.p_a + a_f n0.p_f + a_v n0.p_v + a_{cc} n0.p_{cc} + t$$

Illustration



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- One equation per dart (8 darts.)

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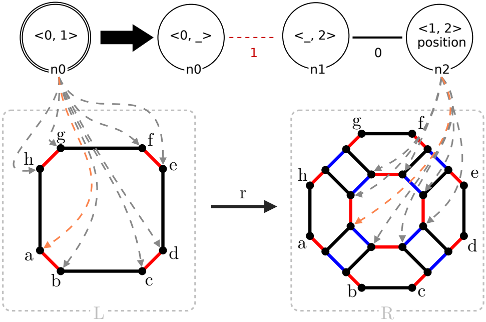
Illustration



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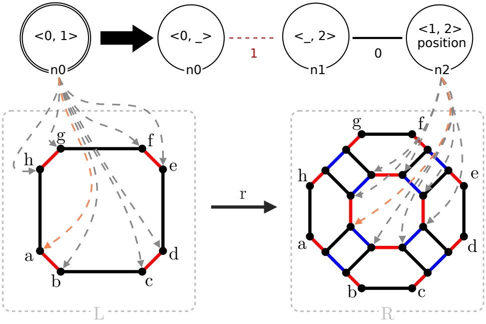

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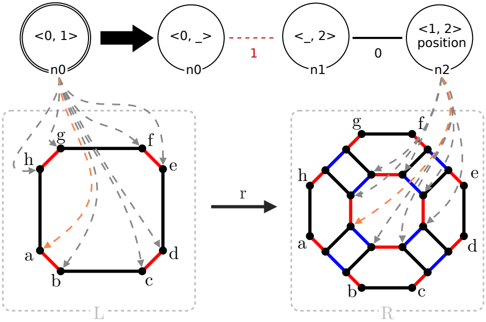

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▶ Solved as a CSP (OR-Tools, Z3)

Solving the barycentric triangulation

- Global equation :

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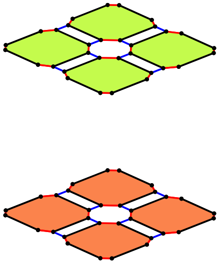
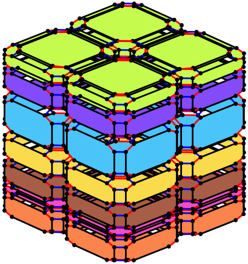
- Solution found :
  - $a_s = -6.601425600620388E-17$
  - $a_a = 0.0$
  - $a_f = 1.0$
  - $a_v = 0.0$
  - $a_{cc} = 0.0$
  - $t = (0.0, 0.0)$

Example in geology (with the geometry)

Before

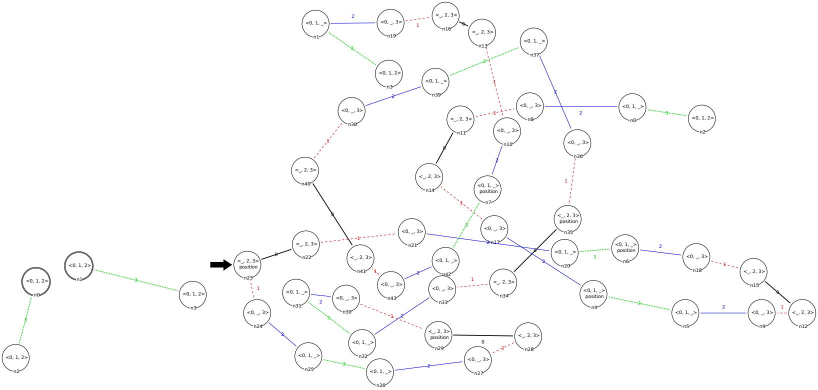


After



Example in geology (with the geometry)

Operation



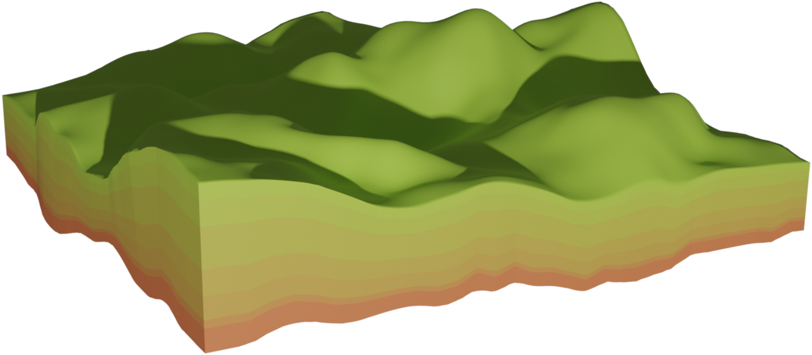
Inference time : $\sim$ 26 ms for the topology,
$\sim$ 549 ms for the embedding expressions

Example in geology (with the geometry)

Before

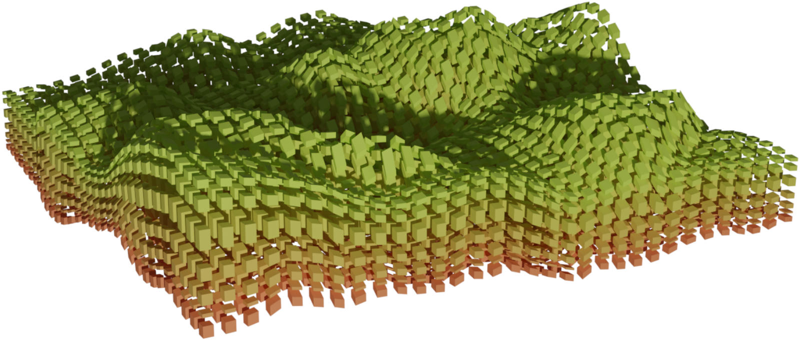
Example in geology (with the geometry)

After



Example in geology (with the geometry)

After



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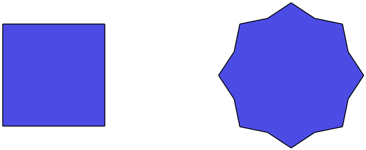

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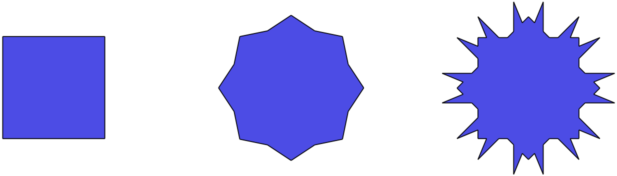

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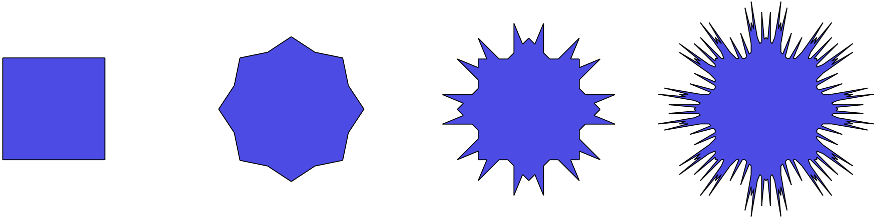
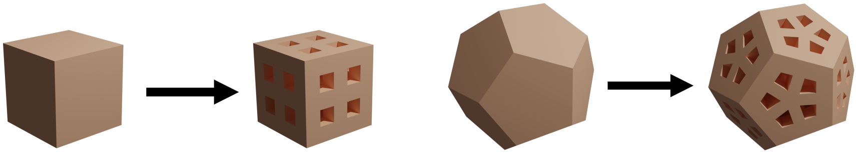


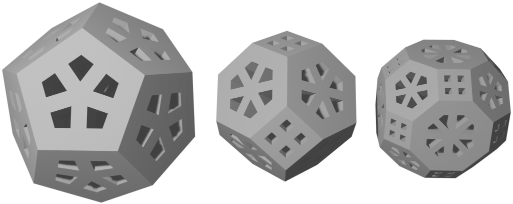
Conclusion

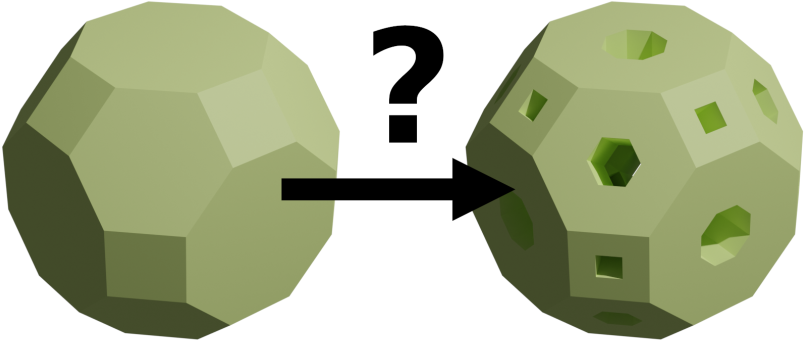
- We presented a method to infer a geometric modeling operation from two instances of an object before and after modification.
- Topology : Graph traversal algorithm.
- Geometry : Affine combinations of points of interests.

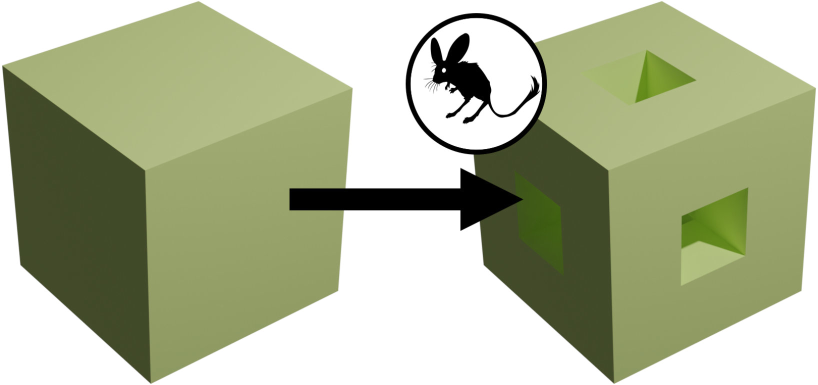


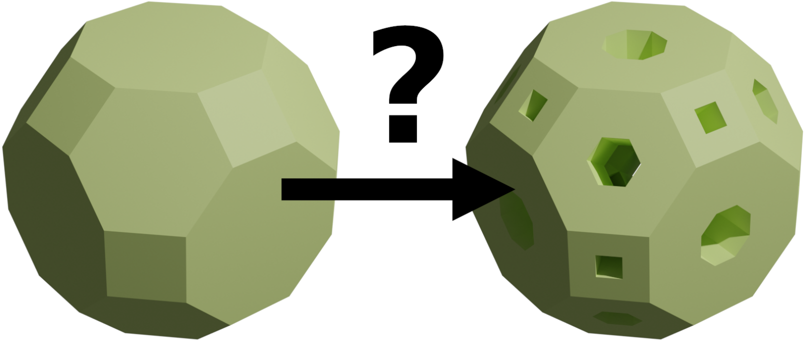
Future works

- Geometric inference :
  - Other points of interest (exploiting the neighboring operator).
  - Other kind of functions (instead of affine combinations).
  - Other embeddings.
- Support in the design of operations
  - Automated generation of instances for a given operation.

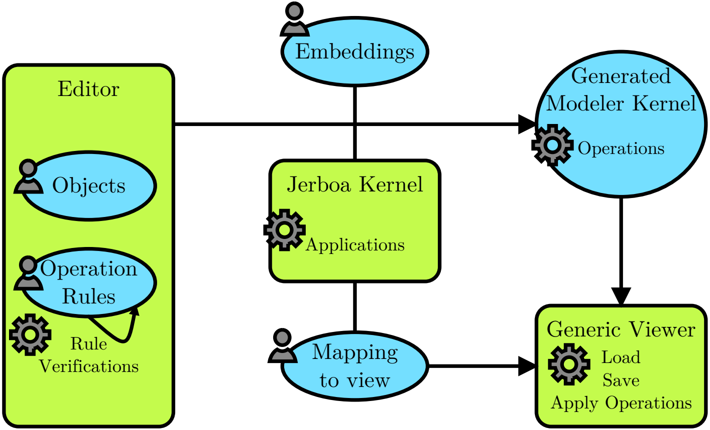








Jerboa's architecture



How to preserve the consistency of the model ?

- Topological constraints : structure,
  - E.g., vertices are incident to edges thats are incident to faces.
- Embedding constraints : geometry,
  - E.g., all elements defining the same vertex should share the same position.

Goal : The modification of a well-formed object should provide a well-formed object.

Rewriting

String rewriting :

- an alphabet Σ
- a set of rewriting rules $u \rightarrow v$ (u and v are words on Σ^*)

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$$\text{sleep} \rightarrow \text{follow}$$

You␣are␣all␣sleeping␣!

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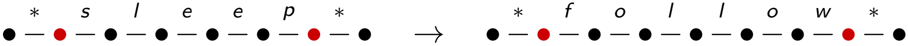
How to rewrite graphs ?

How to rewrite graphs ? (based on [Ehrig 1979])



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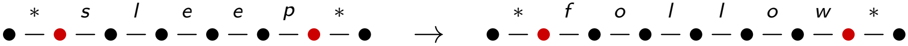

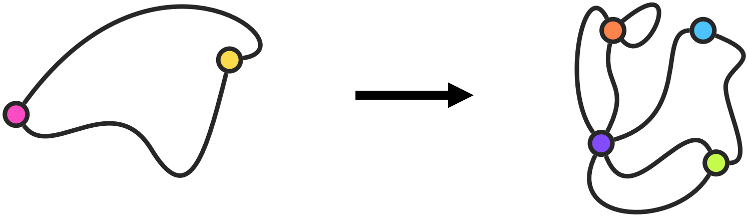

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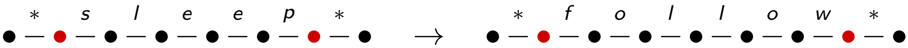
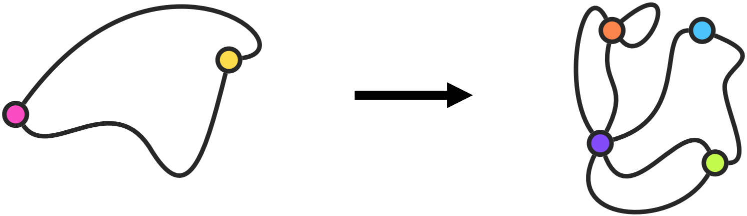


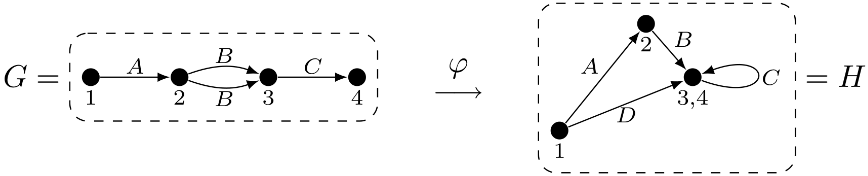


- No notion of beginning and end in a graph.

→ Identify the "gluing" elements.

How to map graphs ? (from [König 18])

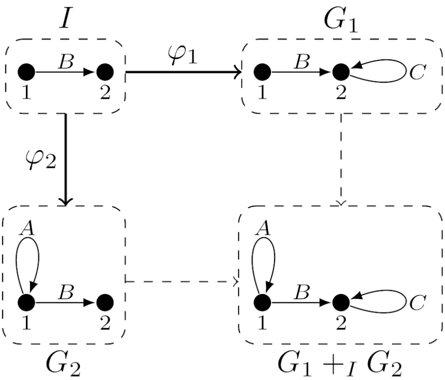
Graph morphism :



- Functions on nodes ands arcs that preserve structure.

How to glue graphs ? (from [König 18])

Graph gluing :



- $\sim$ Quotiented disjoint union.

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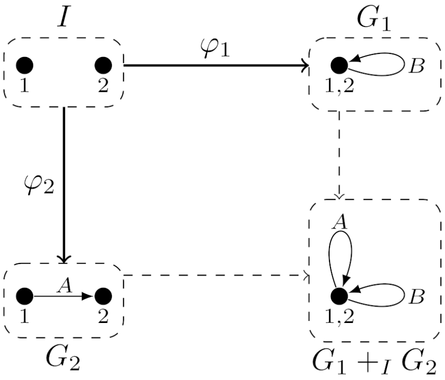

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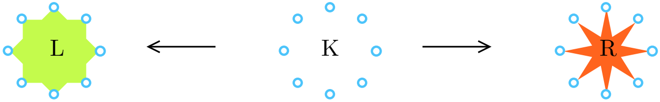
Back to graph transformation rules

- L, K, R, G, D, H are graphs.
- Arrows are graph morphismses.
- Squares are graph gluings.



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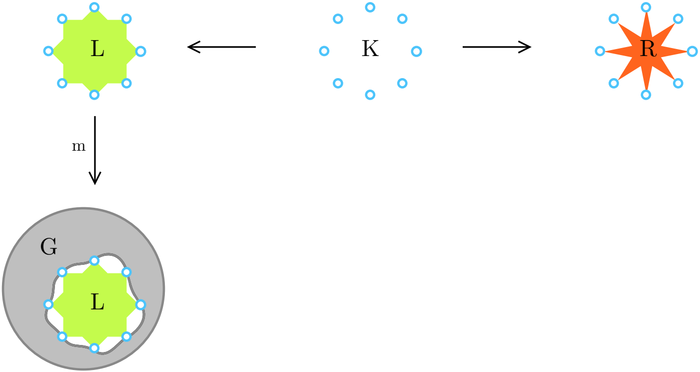

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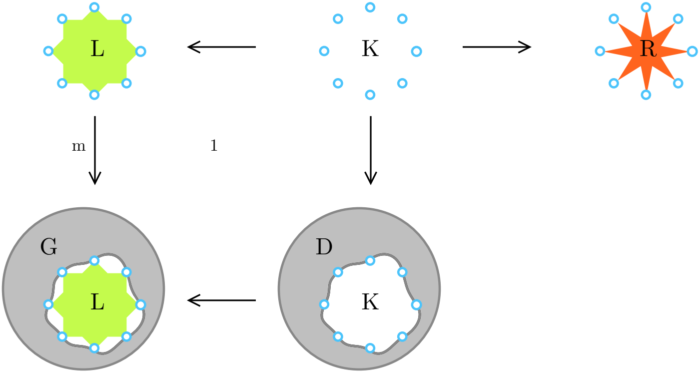

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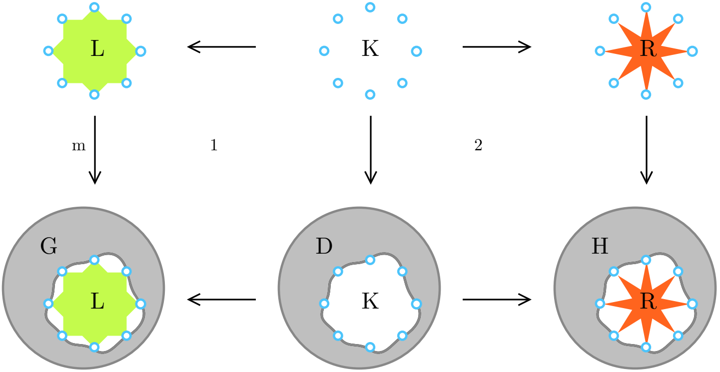
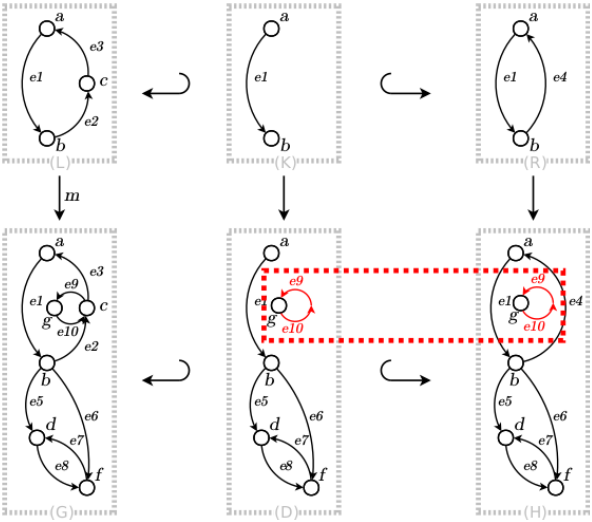


Why does it has to be so complicated ?



Rewriting G-maps

- Most conservative framework (all morphisms are injectives).